Assignment - 2

9/10 Kaur 11/10/24

Name : Raushan Kumar
CRN : 2221139
URN : 2203751
Sec : IT (B2)

Q1. In how many of the distinct permutations of the letters in MISSISSIPPI so that four I's not come together?

ans ⇒ for the distint permutation word MISSISSIPI

I → 4   M → 1
S → 4
P → 2

So

I's ~~not~~ come together = ~~I's come~~
(Total arrangement) − (I's come together)

(i) total arrangement = $\frac{11!}{4! \times 4! \, 2!}$ ⇒ $\frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 4! \times 2!}$

$\frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1 \times 2 \times 1} = 11 \times 10 \times 9 \times 7 \times 5 = 34650$

(ii) No of ways to arrange the ~~wor~~ letter so that I's come together =
letter → MISSISSIPPI. n = 11
I's = 4

⇒ $\frac{8! \times 4!}{4! \times 2! \times 4!}$ ⇒ $\frac{8 \times 7 \times 6 \times 5 \times 4!}{4! \times 2!} = 20160$

$\frac{8\times7\times6\times5}{2!} \Rightarrow 8\times7\times3\times5 = 8\times7\times15 = 8\times105 = 840$

So that, I's not come together $= 34650 - 840 = \underline{33810}$

Q2. In a small village, there are 97 families of which 56 families have at most 2 children in a rural development program, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?

Sol^n: Total families in a small village $= 97$

families have at most 2 children : 56

families have not at most 2 children : 41

we have to choose 20 families in which at least 18 families have at most 2 children :

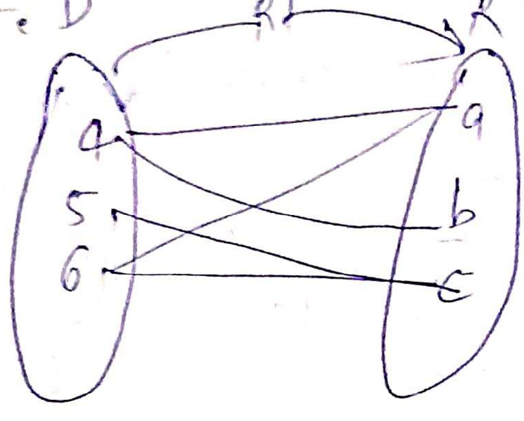
$$\boxed{{}^{56}C_{18} \cdot {}^{41}C_{2} + {}^{56}C_{19} \cdot {}^{41}C_{1} + {}^{56}C_{20} \cdot {}^{41}C_{0}}$$

Q3. Let $X = \{4, 5, 6\}$, $Y = \{a, b, c\}$ and $Z = \{l, m, n\}$, Consider the relation R_1 from X to Y and R_2 from Y to Z.

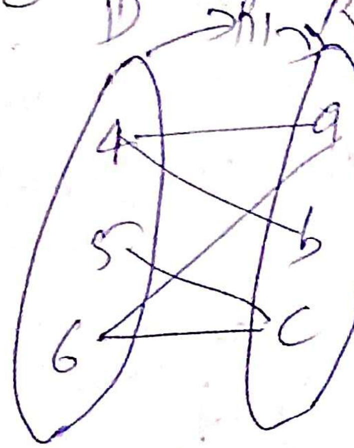
$R_1 = \{(4,a)(4,b)(5,c),(6,a)(6,c)\}$, $R_2 = \{(a,l)(a,n)(b,l)(b,m)$
$(c,l)(c,m)(c,n)\}$

Find (a) $R_1 \circ R_1$ (b) $R_1 \circ R_2$ (c) $R_2 \circ R_1$ (d) $R_2 \circ R_2$

(a) R1oR1 :



@(2)



Relation between R1 and R1 is possible if domain of R1 is equal range of R1.

given R1 = {(4,a),(4,b)(5,c)(6,a)(6,c)}

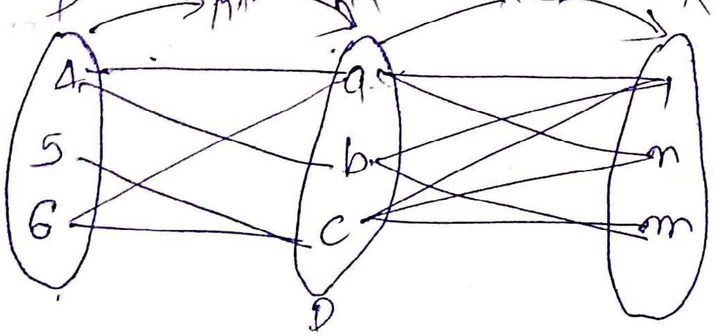
Since ~~Domain~~ Range of R1 is not equal to Domain of R1 so there will be no any relation

ans ⇒ ϕ

(b) R1oR2 ⇒

R1 = {(4,a)(4,b)(5,c)(6,a)(6,c)}

R2 = {(a,l)(a,m)(b,l)(b,m)(c,l)(c,m)(c,n)}

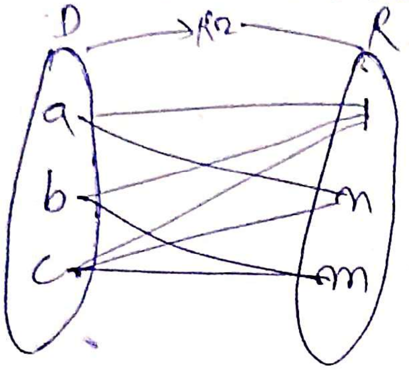


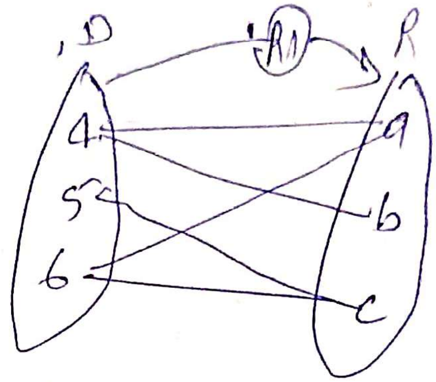
R1oR2 = {(4,l)(4,n)(4,m)(5,l)(5,n)(5,m)(6,l)(6,n)(6,m)}

(c) R2 o R1 →

R2 = {(a,l)(a,n)(b,l)(b,m)(c,l)(c,m)(c,n)}

R1 = {(4,a)(4,b)(5,c)(6,a)(6,c)}



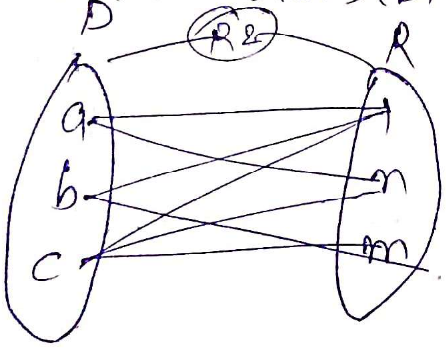


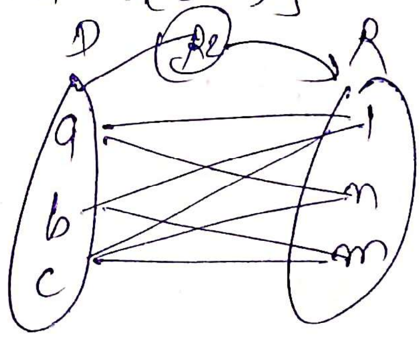
Since, Range of R2 is not equal to Domain of R1 so that there will not be any relation b/w R2 o R1

relation = ϕ

(d) R2 o R2 ⇒

R2 = {(a,l)(a,m)(b,l)(b,m)(c,l)(c,m)(c,n)}





Since, Range of R2 is not equal to Domain of R2 so that there will not be any relation b/w R2 o R2

Relation = ϕ

Q9. Define the following terms with suitable examples
(a) Inclusion - Exclusion principle
(b) partial order relations
(c) pigeonhole principle
(d) Hashing function

(a) Inclusion - Exclusion principle: The Inclusion - Exclusion principle is a method used in combinatorics to calculate the size of the union of multiple sets. It accounts for over-counting by subtracting the sizes of intersections b/w sets. formula

Example: suppose you have two sets A and B:

$n|A| = 10$, $n|B| = 8$, $n|A \cap B| = 3$

then $n(A \cup B) = ?$

Inclusion - Exclusion principle formula is:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = 10 + 8 - 3 = 15$

(b) partial order relation: A relation are on set S is called a partial order relation if R is reflexive, antisymmetric and transitive.

(i) Reflexive: $\forall$ element $a \in A$, $(a,a) \in R$

(ii) Antisymmetric: if $(a,b) \in R$ and $(b,a) \in R$ then $a = b$

(iii) Transitive: if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$

Example: $A = \{1,2,3\}$

$R = \{(1,1)(2,2)(3,3)(1,2)(1,3)(2,3)\}$

the given example is a partial order relation because

(i) reflexive: (1,1)(2,2)(3,3) belongs to R.

(ii) Antisymmetric: There are no pairs of the form (a,b) and (b,a) where $a \neq b$ so, the relation is antisymmetric

(iii) Transitive: The relation satisfies transitivity because (1,2) and (2,3) imply (1,3) which is present in the relation. There are no other pairs that voilate transitivity.

Since, the relation satisfies all three properties, it is a partial order relation.

(c) Pigeonhole principle: The pigeonhole principle states that if n items are put into m containers and n > m, then at least one container must contain more than one item.

Example: If you have 10 socks and 9 drawers, at least one drawer must contain more than ~~one~~ one sock. This principle is useful in ~~providing~~ proving existing result in mathematics.

(d) Hashing function: A hashing function is a function that takes an input (or key) and return a fixed-size string or number, which is typically used to index data in a hash table.

Example: Keys: 064212848, 037149212, 107405723

Assume $h(K) = K \% 111$

(i) 064212848 % 111 = 14

(2) 037149212 % 111 = 65

(iii) 107405723 % 111 = 14

0	
	⋮
14	064212848
15	037149212
	⋮
65	037149212

Q5. In a survey of 500 students of a college, it was found that 49% liked watching football, 53% liked watching hockey and 62% liked watching basketball. Also, 27% liked watching football and hockey both, 29% liked watching basketball and hockey both and 28% liked watching football and basket ball both 5% liked ~~wat~~ watching none of the games.

(i) How many students like watching all the three games?

(ii) find the ratio of number of students who like watching only football to those who like watching only hockey.

(iii) find the no of students who like watching only one of the three given games.

(iv) find the number of students who like watching at least two of the given games.

1 ans ⇒ Let Total No of Students = 500 ⇒ $n(U)$

$n(F)$ = No of students who like watching football

$n(H)$ = No of students who like watching Hockey.

$n(B)$ = No of Students who like basketball.

$n(F \cap H)$ = No of students who like both football and Hockey.

$n(H \cap B)$ = No of students who like both Hockey and Basketball.

$n(F \cap B)$ = No of students who like both football and Basketball.

$n(F \cap H \cap B)$ = No. of students who like all three games.

N = 5% of Students like none of these games.

$n(U) = 500$

$n(F) = 500 \times \frac{49}{100} = 245$

$n(H) = \frac{53}{100} \times 500 = 265$

$n(B) = 62\% \Rightarrow \frac{62}{100} \times 500 = 310$

$n(F \cap H) = \frac{27}{100} \times 500 = 135$

$n(B \cap H) = \frac{29}{100} \times 500 = 145$

$n(F \cap B) = \frac{28}{100} \times 500 = 140$

Given

$n(U) = 500$

$n(F) = 49\%$

$n(H) = 53\%$

$n(B) = 62\%$

$n(F \cap H) = 27\%$

$n(B \cap H) = 29\%$

$n(F \cap B) = 28\%$

$N = \frac{5}{100} \times 500 = 25$

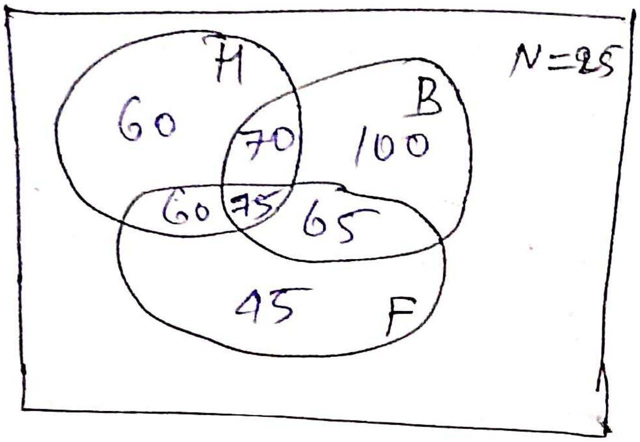
$\therefore n(U) - N = n(F \cup B \cup H)$

$\Rightarrow 500 - 25 = 475 \Rightarrow n(F \cup B \cup H)$

from inclusion - exclusion principle formula:

$$n(F \cup H \cup B) = n(F) + n(H) + n(B) - n(F \cap H) - n(B \cap H) - n(F \cap B) + n(F \cap H \cap B)$$

$$475 = 245 + 265 + 310 - 135 - 145 - 140 + n(F \cap H \cap B)$$



$n(F \cap H \cap B) = 475 - 400$

$n(F \cap H \cap B) = 75$

(i) No of student that like all the three games

$n(F \cap B \cap H) = 75$

(ii) Ratio of No of stu who like watching only football to those who like watching only hockey $= \frac{45}{60} = \frac{15}{20} = \frac{3}{4}$

(iii) Student who like only one of the three given gam $= 60 + 100 + 45 = 205$

(iv) No of student who like at least two of the games

$60 + 65 + 70 + 75 = 270$ students.

Q6. Examine the usage of recurrence relations and solve:

$t_n = 4(t_{n-1} + t_{n-2})$ where $t_n = 1$ if $n=0$ and $n=1$

Usage of Recurrence Relations

(i) **Dynamic programming:** Recurrence relations from the basis of dynamic programming solutions, where problems are broken down into smaller subproblems. For example the fibonacci sequence can be defined using the recurrence relation:

$$f(n) = f(n-1) + f(n-2)$$

		$n-2$	$n-1$	n
0	1	1	2	3

$$f(n) = f(n-1) + f(n-2)$$

$$f(3) = f(2) + f(1)$$

$$f(3) = 2+1 = 3$$

$$\boxed{f(3)=3} \quad \boxed{f(n)=3}$$

(ii) **Divide and conquer:** Many divide and conquer algorithms like Zuick short or matrix multiplication, use reccurence relations to describe their behavior.

Ex → $T(n) = 7T\left(\frac{n}{2}\right) + O(n^2)$

(iii) **Counting problems:** Reccurrence relations are used to count combinational objects. for ex the no of ways to climb stairs where one can take 1 or 2 steps at a time can be modeled as

$C(n) = C(n-1) + C(n-2)$ This is simmilar to fibonacci series.

Algorithm Analysis: Recurrence relations are commonly used to describe the time complexity of recursive algorithms for example the time complexity of the merge sort algorithm can be described using the reccurrence relation.

$$T(n) = 2T(\frac{n}{2}) + O(n)$$

where $T(n)$ is the time complexity for input size n.

$t_n = 4(t_{n-1} + t_{n-2})$ where $t_n = 1$ if $n = 0$ and $n = 1$

$t_n = 4t_{n-1} + 4t_{n-2}$

$t_n - 4t_{n-1} - 4t_{n-2} = 0$

It is homogeneous.

Step 1: put $t_n = r^n$, $t_{n-1} = r^{n-1}$, $t_{n-2} = r^{n-2}$

$r^n - 4r^{n-1} - 4r^{n-2} = 0$ Step 2: find an eq^n in terms of r^n. This is called characteristic eq^n.

$r^n\left[1 - \frac{4}{r} - \frac{4}{r^2}\right] = 0$

$r^2 - 4r - 4 = 0$ Step 3: find out root of the characteristic eq^n :⇒

$x = \frac{-(b) \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - (4 \times 1 \times -4)}}{2 \times 1}$

$x = \frac{4 \pm \sqrt{16 + 16}}{2} = \frac{4 \pm \sqrt{32}}{2}$

2	32
2	16
2	8
2	4
	2

$x = \frac{4 \pm 4\sqrt{2}}{2}$ $x = \frac{4 + 4\sqrt{2}}{2}, \frac{4 - 4\sqrt{2}}{2}$

$x = 2+2\sqrt{2}$, $x = 2-2\sqrt{2}$

Since two roots are distinct then the general solution will be:

$$a_n = b_1 r_1^n + b_2 r_2^n$$

Since the initial conditions are $t_0 = 1$, $t_1 = 1$

for $a_0 = 1$

$$a_0 = b_1 r_1^0 + b_2 r_2^0$$

$$1 = b_1 + b_2 \quad \text{— (1)}$$

for $a_1 = 1$

$$a_1 = b_1 r_1^1 + b_2 r_2^1$$

$$1 = b_1(2+2\sqrt{2}) + b_2(2-2\sqrt{2}) \quad \text{— (2)}$$

from eq^n (1)

$$b_2 = 1 - b_1 \quad \text{— (3)}$$ put eq (3) in (2)

$$1 = b_1(2+2\sqrt{2}) + [(1-b_1)(2-2\sqrt{2})]$$

$$1 = b_1(2+2\sqrt{2}) + [2 - 2\sqrt{2} - 2b_1 + 2\sqrt{2}b_1]$$

$$1 = b_1(2+2\sqrt{2}) + 2 - 2\sqrt{2} - 2b_1 + 2\sqrt{2}b_1$$

$$1 = 2b_1 + 2\sqrt{2}b_1 + 2 - 2\sqrt{2} - 2b_1 + 2\sqrt{2}b_1$$

$$1 = 4\sqrt{2}b_1 + 2 - 2\sqrt{2}$$

$$1 - 2 + 2\sqrt{2} = 4\sqrt{2}b_1$$

$$-1 + 2\sqrt{2} = 4\sqrt{2}b_1$$

$$4\sqrt{2}b_1 = 2\sqrt{2} - 1$$

$$b_1 = \frac{2\sqrt{2}}{4\sqrt{2}} - \frac{1}{4\sqrt{2}}$$

$$b_1 = \frac{1}{2} - \frac{1}{4\sqrt{2}}$$

$$b_1 = \frac{2\sqrt{2}-1}{4\sqrt{2}} \quad \text{— (4)}$$

put eqⁿ (4) in eqⁿ (1)

$$b_1 + b_2 = 1$$

$$\left(\frac{2\sqrt{2}-1}{4\sqrt{2}}\right) + b_2 = 1$$

$$b_2 = 1 - \left(\frac{2\sqrt{2}-1}{4\sqrt{2}}\right)$$

$$b_2 = 1 - \left[\left(\frac{2\sqrt{2}}{4\sqrt{2}} - \frac{1}{4\sqrt{2}}\right)\right]$$

$$b_2 = 1 - \left[\frac{1}{2} - \frac{1}{4\sqrt{2}}\right]$$

$$b_2 = 1 - \frac{1}{2} + \frac{1}{4\sqrt{2}}$$

$$b_2 = \frac{1}{2} + \frac{1}{4\sqrt{2}} = \frac{2\sqrt{2}+1}{4\sqrt{2}}$$

$$\boxed{b_2 = \frac{2\sqrt{2}+1}{4\sqrt{2}}} \quad \boxed{b_1 = \frac{2\sqrt{2}-1}{4\sqrt{2}}}$$

put b_1, b_2 value in general Solution

$$\boxed{a_n = \left(\frac{2\sqrt{2}-1}{4\sqrt{2}}\right)(2+2\sqrt{2})^n + \left(\frac{2\sqrt{2}+1}{4\sqrt{2}}\right)(2-2\sqrt{2})^n}$$

ans