Assignment - 1

Discrete Math

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Section : IT(B2)

Q1. Let R and S be the following relations on $A = \{1,2,3\}$

$R = \{(1,1)(1,2)(2,1)(3,1)(3,3)(2,3)\}$, $S = \{(1,2)(1,3)(2,1)(3,3)\}$

find:

(a) $R \cup S$

(b) R^c

(c) $R \circ S$

(d) $S^2 = S \circ S$

(e) $R - S$

(f) $R \oplus S$

(a) $R \cup S = \{(1,1)(1,2)(1,3)(2,1)(2,3)(3,1)(3,3)\}$

(b) $R^c = \underline{A \times A}$ that are not in R

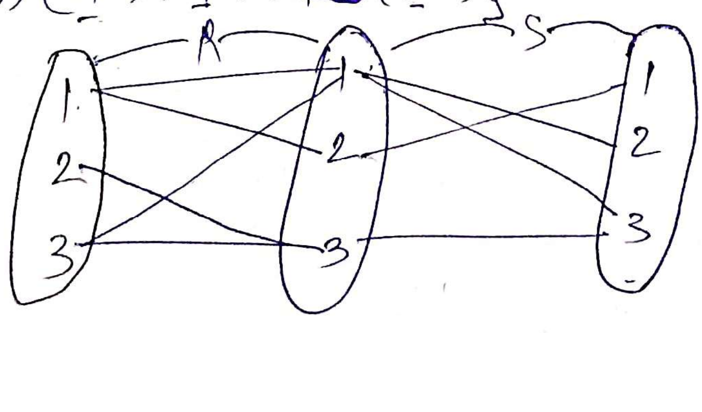
$A \times A =$

$\{(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)\}$

~~$R^c = A \times A -$~~

$R^c = \{(1,3)(2,1)(2,2)(3,2)\}$.

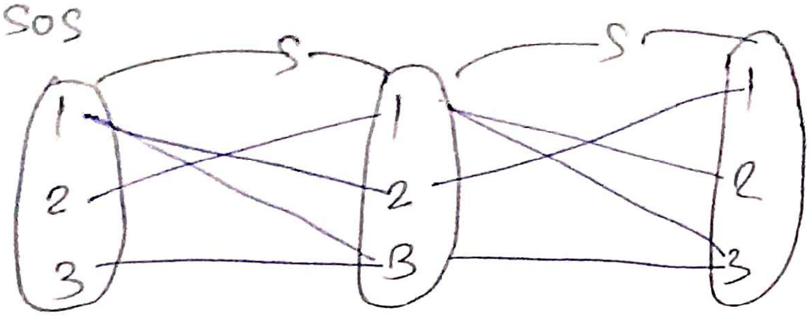
(c) $R \circ S \rightarrow$



(d) $R \circ S = \{(1,2)(1,3)(1,1)(3,2)(3,3)(2,3)\}$

~~Q2. A survey of fa~~

$S^2 = S \circ S$



$S \circ S = \{(1,1)(1,3)(2,2)(2,3)(3,3)\}$

(e) $R - S = \{(1,1)(1,2)(2,3)(3,1)(3,3)\} - \{(1,2)(1,3)(2,1)(3,3)\}$

$R - S = \{(1,1)(2,3)(3,1)\}$

(f) $R \oplus S = (R \cup S) - (R \cap S)$

$R \cup S = \{(1,1)(1,2)(1,3)(2,1)(2,3)(3,1)(3,3)\}$

$R \cap S = \{(1,2)(3,3)\}$

$R \oplus S = \{(1,1)(1,2)(1,3)(2,1)(2,3)(3,1)(3,3)\} - \{(1,2)(3,3)\}$

$R \oplus S = \{(1,1)(1,3)(2,1)(2,3)(3,1)\}$

Q2. A survey of faculty and students at a school revealed the following information 51 admire Maths, 49 admire language, 60 admire craft, 34 admire maths, and languages, 32 admire language and craft, 36 admire Math and craft, 24 admire all three of the courses, 1 admire none of three courses.

12. A survey of faculty and student at a school revealed the following information: 51 admire maths, 49 admire language, 60 admire craft, 34 admire maths and language, 32 admire language and craft, 36 admire maths and craft, 24 admire all three of the course, 1 admire none of the three courses.

(a) How many admire craft, but not language not maths?

(b) How many admire exactly one of the courses?

(c) How many admire exactly two of the courses

(d) How many admire all three?

~~Ans~~

Number of people who admire maths $= M$

" " " " " language $= L$

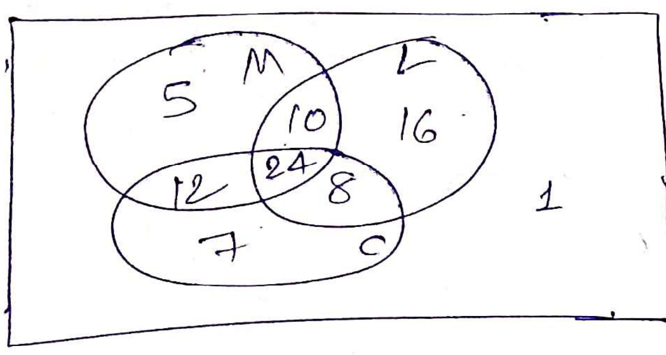
" " " " " craft $= C$

Number of peo who admire none of the courses $= N$

$M = 51$ | $M \cap L = 34$ | $n(M \cap C) = 36$

$L = 49$ | $L \cap C = 32$ | $\boxed{N = 1}$

$C = 60$ | $M \cap L \cap C = 24$



Extra question

(i) how many admire craft but not lang.

$\Rightarrow 12 + 7$

(ii) how many admire craft but not math

$\Rightarrow 7 + 8 = 15$

(a) Admire craft only : $n(C) - n(M \cap C) - n(C \cap L) + n(M \cap L \cap C)$

~~$- 12 - 16 + 24$~~ $=$

$\Rightarrow 60 - 36 - 32 + 24 = 16$ People.

(b) Admire only maths $= M - n(M \cap L) - n(M \cap C) + n(M \cap L \cap C)$

$51 - 34 - 36 + 29 = 5$ people

Total

Admire only Language $= n(L) - n(M \cap L) - n(L \cap C) + n(M \cap L \cap C)$

$= 49 - 34 - 32 + 29 = 7$ People.

Admire only craft $= n(C) - n(M \cap C) - n(L \cap C) + n(M \cap L \cap C)$

$= 60 - 36 - 32 + 24 = 16$ people

Total who admire exactly one course $= 5 + 7 + 16 = 28$ people

(c) How many admire exactly two of the courses.

Admire exactly Maths and L $= n(M \cap L) - n(M \cap L \cap C) = 34 - 24 = 10$

" " L and C $= n(L \cap C) - n(M \cap L \cap C) = 32 - 24 = 8$

" " M and C $= n(M \cap C) - n(M \cap L \cap C) = 36 - 24 = 12$

Total who admire exactly two courses $= 10 + 8 + 12 = 30$ peopl

(d) How many admire all three?

Admire all three courses $= n(M \cap L \cap C) = 24$ People.

Q3. From a survey of 120 People, the following data was obteined: 90 ~~own~~ owned a car, 35 owned a computer, 40 owned a house, 32 owned a car and a house, 21 owned a house and a computer, 26 owned a car and a computer, 17 owned all the three facilities.

(i) How many people owned neither of the three?

(ii) How many people owned only a car?

(iii) How many people owned only a computer?

Total people surveyed $m(U) = 120$

No of people who own a car $m(C) = 90$

No of people who own a computer $m(P) = 35$

no of people who own a House $m(H) = 40$

$m(C \cap H) = 32$ | ~~m(P∩H)~~ $m(C \cap P) = 26$

$m(H \cap P) = 21$ | $m(C \cap P \cap H) = 17$

(i) how many people owned neither of the three?

$m(C \cup P \cup H) = m(C) + m(P) + m(H) - m(C \cap P) - m(P \cap H) - m(H \cap C) + m(C \cap P \cap H)$.

$m(C \cup P \cup H) = 90 + 35 + 40 - 26 - 21 - 32 + 17 = 103$

so, the no of people who owned neither of three.

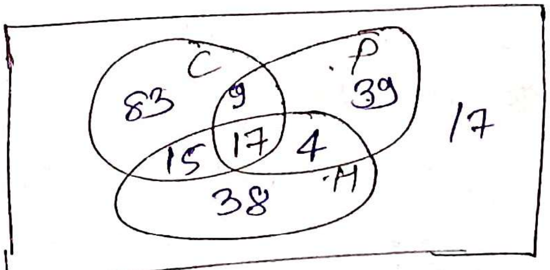
$m(U) - m(C \cup P \cup H) = 120 - 103 = 17$ people.

(ii) How many people owned only a car?

only car $\Rightarrow m(C) - m(C \cap P) - m(C \cap H) + m(C \cap P \cap H)$

$90 - 26 - 32 + 17$

$32 + 17 = 49$ people



(iii) How many people owned only a computer?

$m(P) - m(C \cap P) - m(H \cap P) + m(C \cap P \cap H)$

$35 - 26 - 21 + 17 = 5$ people

Q4. Give an example of relation which is symmetric but neither reflexive nor anti symmetric nor transitive.

Symmetric relation : property: A Relation on a set where $(a,b) \in R$ and $(b,a) \in R$

Anti - symmetric : A Relation on a set A where $(a,b) \in R$
$(b,a) \in R$
and $(a=b)$

Reflexive relation : A Relation on a set A if aRa and $\forall\ a \in A$

Transitive relation : A Relation on a set A if
$(a,b) \in R$
$(b,c) \in R$
$(a,c) \in R$

Ex → $\{(1,2)(2,1)(3,1)(1,3)\}$

Qs. Determine ~~whether~~ whether the following are relations, ~~reflexive~~ reflexive, symmetric, ~~transitive~~ transitive

(i) $A = \{2, 3, 4\}$

$R = \{(2,2)(3,3)(4,4)(2,3)(3,4)\}$

(ii) $R = \{(x,y) : y = x+5$ & $x<4 ; x,y \in R\}$

(i) reflexive : since $(a,a) \in R$, $a \in A$ it is reflexive

- symmetric : $(a,b) \in R$ but $(b,a) \notin R$
- transitive : $(a,b) \in R$, $(b,c) \in R$ but $(a,c) \notin R$

(2) $R = \{(x,y) : y = x+5$ & $x<4 ; x,y \in R$

Sol:
$x=0 \quad y=5 \mid x=2 \quad y=7$
$x=1 \quad y=6 \mid x=3 \quad y=8$

$R = \{(0,5)(1,6)(2,7)(3,8)\}$

(i) reflexive: aRa and $\forall a \in A$

It is not reflexive

(ii) Symmetric Relation: since $a, b \in R$ but $b, a \notin R$

It is not symmetric Rela.

(iii) transitive: since $(a,b) \in R$, $(b,c) \notin R$ and ~~ate~~ $(a,c) \notin R$.

It is not transitive.

Q6. prove De Morgan's law using an example.

Let $U = \{1,2,3,4,5,6,7,8\}$

$A = \{3,4,5\}$

$B = \{4,5,6\}$

~~For~~ we know that Demorgan's law $= \underset{L.H.S}{(A \cup B)'} = \underset{R.H.S}{A' \cap B'}$

$(A \cup B) = \cancel{\{4,5\}} \{3,4,5,6\}$

$(A \cup B)' = U - (A \cup B) \Rightarrow \{1,2,3,4,5,6,7,8\} - \{3,4,5,6\}$

$(A \cup B)' = \{1,2,\cancel{3},\cancel{4},7,8\}$

from R.H.S $\Rightarrow A' \cap B' = A' = \{1,2,3,4,5,6,7,8\} - \{3,4,5\}$

$A' = \{1,2,6,7,8\}$

$B' = \{1,2,3,4,5,6,7,8\} - \{4,5,6\}$

$\Rightarrow B' = \{1,2,3,7,8\}$

$A' \cap B' = \{1, 2, 6, 7, 8\} \cap \{1, 2, 3, 7, 8\}$

$A' \cap B' = \{1, 2, 7, 8\}$

$L.H.S = \{1, 2, 7, 8\}, R.H.S = \{1, 2, 7, 8\}$

Since $L.H.S = R.H.S$

Hence proved $\boxed{(A \cup B)' = A' \cap B'}$