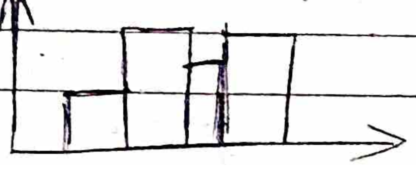
* Differentiate between Discrete and Continuous value.

Aspect	Discrete value	continuous values
Definition	values that are distinct or seprate	values that can take any value within a given range
Nature	Countable	uncountable
Represen tation	points on Graph	line or curve on graph
Example	integers, no of students in a class	Real no, time, temperature, height
possible values	specific, finite set of values	any values within a given interval

Discrete Mathematics

Definition: Discrete mathematics is the study of mathematics structures that are countable or otherwise distinct and seprable.
Discrete value → Histogram graph, line

continuous values are measurable whereas discrete values are ~~contable~~ countable.



Set theory

The collection of well defined distinct objects is known as a set.
Ex → the collⁿ of children in class 7 whose weight exceeds 35kg represent a set. Set of vowels in english alphabet

Notation: A set is usually denoted by capital letters and the element are denoted by small letters

→

The different objects that form a set are called the elements of a set

The elements of a set are written in any order and are not replaced.

The change in order of writting the elements doesn't make any changes in the set $\{a, b, c, d\} = \{b, d, c, a\}$

if one or many elements of a set are replaced, the set remains the same.
$U = \{$letters of word committee$\}$
$U = \{C, O, M, \cancel{M}, I, \cancel{T}, T, E\}$

(i) Algorithm and Data structure:
Application: Efficiently organizing & processing data in computer programs.
Example: sorting, searching or traversal

(ii) Cryptography: Securing communication & data through encryption techniques
Example: RSA Algorithm for secure data transmission.

(iii) logic and Boolean Algebra:
Application → Designing & optimizing digital circuits and understanding programming language semantics.

Example: logic gates in computer processors and digital circuits design

(iv) Network theory: Modeling and analyzing networks such as the internet social networks and transportation system.

(v) Combinatorics: Counting problems: Determining the no of ways certain events can occur.

(vi) Design theory: Creating experimental design & coding theory.

Representation of Set ÷

1. Statement form
2. Roster form
3. Set Builder form

(1) In Roster form all the elements of the Set are ~~and~~ Separated by Commas and enclosed between the curly brases.

$K = \{1, 2, 3, 4\}$

(2) Set builder form : in this all the elements have a common property

$A = \{A, D, R, E, S\}$

Q. Slove using the 3 methods of representation of a Set.

1. A two digit perfect square nos.
2. Set having all the elements which are even prime no.

(1) ans ⇒ $K = \{4, 5, 6, 7, 8, 9\}$
$= \{16, 25, 36, 49, 64, 81\}$ $n(A) = 6$

Set builder $= K = \{x : x$ is a two digit perfect square$\}$

(2) ans ⇒ Roster $A = \{2\}$ — singleton Set
Set but $A = \{x / x$ is a even prime no$\}$

Circle O come in vowel

Cardinality of Set $n(A)$ = no of elements in a Set ↳ denoted

$\in$ = belong to
$\notin$ = Not belong to
/ or : Such that
ϕ → Null
$n(A)$ → Cardinality
$\cup$ → union
$\cap$ → intersection
$A \subset B$ → A is subset of B, proper subset
$A \supset B$ → superset proper
$A \supseteq B$ → only superset
$A = B$ ⇒ equal set

Ex ⇒ $A = \{1, 2, 3\}$
$n(A) = 3$

finite Set : A Set which contains a definite no of elements is called a finite set

Infinite Set ⇒ A Set having ~~same~~ infinite no of elements ~~A = {p, q, r, s}~~ ~~B = {q, r, s, p}~~

$A = \{1, 2, 3, \ldots\}$ $\{N = 1, 2, 3 \ldots\}$

equivalent Set ⇒ $A = \{p, q, r, s\}$ / Disjoint
$B = \{a, b, c, d\}$

overlapping Set : $A = \{a, b, c\}$
$B = \{a, e, i, o, u\}$

power set : The collection of all the subsets of Set A is called the power Set of A. it is denoted by $P(A)$.

$A = \{1, 2, 3\}$

$P(A) = \{1\}, \{2\}, \{3\}, \{1,2,3\}, \{1,2\}, \{2,3\}$
$\{1,3\}, \phi = 8 \text{ or } 2^3$

$\boxed{n(A) = 2^n}$

(*) universal set: A set which contains all the element of the other given set, A universal set is a set that includes all the elements or objects of other set as well as its own

$A = \{1, 2, 3\}, \; B = \{a, b, c\}$

$U = \{1, 2, 3, a, b, c\}$

* Operation on set:

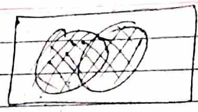
Difference
union
intersection
cartesian product
complement
symmetric difference

union:

* union of the sets ~~and~~ A and B is defined to be the set of all those elements which belong to A or B

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

$A = \{1, 2, 3, 4\}$
$B = \{3, 4, 5, 6, 7\}$

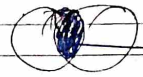


$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

Intersection: Intersection of two sets A and B is the set of all those elements which belong to both the sets A and B

Ex→ $A \cap B = \{x : x \in A \text{ and } x \in B\}$

$A = \{1, 2, 3, 4\}$
$B = \{3, 4, 5, 6, 7\}$ $\quad A \cap B = \{3, 4\}$

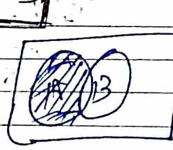

$\rightarrow A \cap B$

* Difference:

The difference of two sets A and B is defined to B. A set of all those elements which belongs to A but do not belong to B and It is denoted by $A - B$ & $A \setminus B$
the set $A-B$ also known as relative complement of B w.r.t A

$\boxed{A - B = \{x : x \in A \text{ and } x \notin B\}}$

Ex→ $A = \{a, b, c, d\}$
$B = \{p, q, b, r\}$

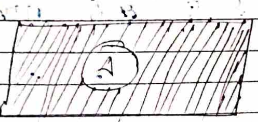


$A - B = \{a, c, d\}$

(*) Complement : The complement of Set A is a set of all those set of (element) universal set which do not belong to A.

A^c or A'

$A' = \{x : x \in U \text{ and } x \notin A\}$



if $U = \{1,2,3,4,5,6,7,8,9\}$

$A = \{2,4,8\}$

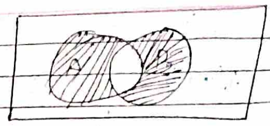
$\boxed{A^c = \{1,3,5,7,9\}}$

(*) Symmetric difference : The symmetric difference of two Set A and B is the set containing all the elements that are in A or in Set B but not in both the Set. It is denoted by ⊕

$\boxed{A \oplus B = (A \cup B) - (A \cap B)}$

$(A \cup B) \setminus (A \cap B)$

$(A \setminus B) \cup (B \setminus A)$



Ex => $x = \{1,2,3,4,5\}$
$y = \{4,5,6,7,8\}$

$A \oplus B = (A \cup B) - (A \cap B)$

$A \oplus B = \{1,2,3,4,5,6,7,8\} - \{4,5\}$

$\boxed{A \oplus B = \{1,2,3,6,7,8\}}$

(*) Cartesian product : If Set A and Set B are the two sets then the Cartesian product of Set A and Set B is a set of all ordered pairs of (a,b) such that a is an element of Set A and b is an element of Set B; It is denoted by A×B.

$A \times B = \{(a,b) : a \in A, b \in B\}$

Ex => $A = \{1,2,3,4\}$, $B = \{x,y,z\}$

$A \times B = \{(1,x),(1,y),(1,z),(2,x),(2,y),(2,z),(3,x),(3,y),(3,z),(4,x),(4,y),(4,z)\}$

* General identities:

$\left.\begin{array}{l} A \cup A = A \\ A \cap A = A \end{array}\right]$ ~~identi~~ idempotent law

* Associative law: $\boxed{(A \cup B) \cup C = A \cup (B \cup C)}$

$\boxed{(A \cap B) \cap C = A \cap (B \cap C)}$

* Commutative law:

~~$(A \cup B) \cup C$~~ = $A \cap B = B \cap A$

$A \cup B = B \cup A$

* Distributive law =

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

* ~~demorg~~ De-Morgan's law:

$(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

* identity laws: $A \cup \phi = A$

$A \cap \phi = \phi$

$A \cap U = A$

$A \cup U = U$

$A \cup A' = U$

$A \cap A' = \phi$

$U' = \phi$

$\phi' = U$

$(A')' = A$ involution law

Q. Let $A = \{1, 2, 4, 5\}$

$B = \{a, b, c, f\}$

$C = \{a, 5\}$

$(A \cup C) \times B$

$A \cup C = \{1, 2, 4, 5, a\} \times \{a, b, c, f\}$

$A \cup C = \{(1,a), (1,b)(1,c)(1,f)(2,a)(2,b)(2,c)(2,f)(4,a)(4,b)(4,c)(4,f)(5,a)(5,b)(5,c)(5,f)(a,a)(a,b)(a,c), (a,f)\}$

Q. If $A = \{1, 2, 5, 6\}$

$B = \{2, 5, 7\}$

$C = \{1, 3, 5, 7, 9\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

find $A \cap B$, $B \cup C$, A', $A - B$, $B - C$, $A \oplus B$, $A \cup C$, $(A \cup C) - B$, $(A \cup B)'$, $(B \oplus C) - A$

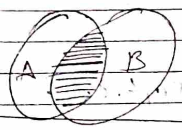
Q. Find the no of subsets and no of proper subsets for the given set $A = \{5, 6, 7, 8\}$

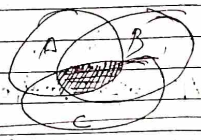
(DM)

* Inclusion exclusion principle:

(i) let A and B be any finite sets, then
$\boxed{n(A\cup B) = n(A) + n(B) - n(A\cap B)}$ — (i)

(ii) for any finite sets A, B, C
$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(A\cap C) - n(B\cap C) + n(A\cap B\cap C)$ — (ii)

(i) 

(ii) 

PYQ

(I) Proper subset: A proper subset of a set A is a set B that contains some but not all elements of A. In other words: Every element of B is also an element of A, but A has at least one element that is not in B. denoted As . $A \subset B$

Ex => $A = \{1, 2, 3, 4\}$

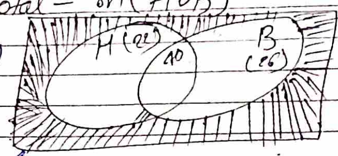
$B = \{1, 2\}$

$A \subset B$

Q In a class of 40 student 22 play hockey 26 play basketball and 14 play both the games how many student don't play either of two games.

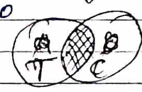
$n(H) = 22$ total : 40
$n(B) = 26$ $n(U) = 40$
$n(H\cap B) = 14$
$n(H\cup B) = \text{total} - n(H\cup B)$

$n(H\cup B) = n(H) + n(B) - n(H\cap B)$
$= 22 + 26 - 14$
$= 48 - 14 = 34$
$\text{total} - n(H\cup B) \Rightarrow 40 - 34 = 6$

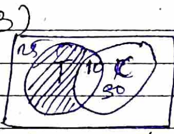


Q In a survey of 60 peoples 25 like tea, 30 like coffee, 10 like both, how many people like only tea.

total = 60
$n(T) = 25$
$n(C) = 30$
$n(T\cap C) = 10$



$\because n(A\cup B) = n(A) + n(B) - n(A\cap B)$
$n(T) - n(T\cap C) =$
$25 - 10 = 15$

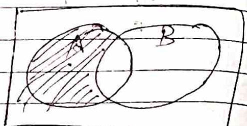


Q. Suppose a list A contains 30 students in a mathematics class and a list B contains 35 students in an english class. suppose there are 20 names on both

Set Qmar

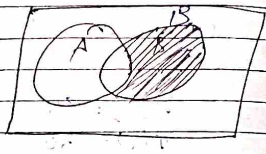
list find no of student
(i) only on list A
(ii) only on list B
(iii) on list A or B or both
~~(iv) on exactly one list~~
(iv) on exactly one list.

(i) $n(A) = 30$
$n(B) = 35$
$n(A \cap B) = 20$



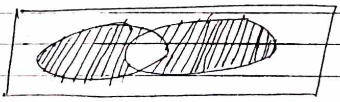
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A) - n(A \cap B) = 30 - 20 = 10$

(ii) $n(B) - n(A \cap B)$
$35 - 20 = 15$



(iii) $(A \cup B) = ?$
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A \cup B) = 30 + 35 - 20$
$= 45 - 20 = 25$

(iv) $n(A) + n(B) = 10 + 15 = 25$



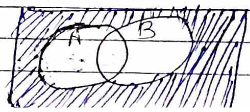
(Q) If set m contains all value of x such that x is a prime no less than 20 and set n contains all the value of x such that x is odd no less than 10 find $(m \cap n)$

$m = \{2, 3, 5, 7, 11, 13, 17, 19\}$
$n = \{1, 3, 5, 7, 9\}$
$(m \cap n)$
$\Rightarrow \{3, 5, 7\}$

(Q) In a group of 50 people 28 have travell to europe, 31 have to asia 10 both the continents how many people have not traveled either in one continent.

ans ⇒ total = 50, $n(E) = 28$, $n(A) = 31$
$n(E \cap A) = 10$

ans = total − $n(E \cup A)$
$n(E \cup A) = n(E) + n(A) - n(E \cap A)$
$= 28 + 31 - 10$
$= 59 - 10 = 49$
ans $= 50 - 49 = 1$



Q. which of these sets are equal
$S_1 = \{x, y, z\}$ $S_2 = \{z, y, z, x\}$
$S_3 = \{y, x, y, z\}$
$S_4 = \{y, z, x, y\}$
all the sets are equal

Q. Let U is equal to $U = \{1, 2, 3, 4 ... 9\}$ be the universal set and let
$A = \{1, 2, 3, 4, 5\}$ $B = \{4, 5, 6, 7\}$
$C = \{5, 6, 7, 8, 9\}$ $D = \{1, 3, 5, 7, 9\}$
$E = \{2, 4, 6, 8\}$ $F = \{1, 5, 9\}$
find $(A \cup B)$, $(A \cap C)$, A', B', E', $A-B$, $(D-E)$, $(C \oplus F)$, $(B \oplus F)$

(i) $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

(ii) $A \cap C = \{5\}$ (iii) $A' = \{6, 7, 8, 9\}$

(iv) $B' = \{1, 2, 3, 8, 9\}$ (v) $E' = \{1, 3, 5, 7, 9\}$

(vi) $A-B = \{1, 2, 3\}$ (vii) $D-E = \{1, 3, 5, 7, 9\}$

(viii) $C \oplus F = (C \cup F) - (C \cap F)$

$= \{1, 5, 6, 7, 8, 9\} - \{5, 9\}$

$C \oplus F = \{1, 6, 7, 8\}$

$B \oplus F = (B \cup F) - (B \cap F)$

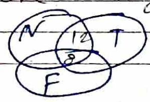
$B \cup F = \{1, 4, 5, 6, 7, 9\}$
$B \cap F = \{5\}$
$B \oplus F = \{1, 4, 5, 6, 7, 9\} - \{5\}$
$\boxed{B \oplus F = \{1, 4, 6, 7, 9\}}$

(IM) Q. In a survey of 120 people it was found that 65 read NW magzine and 45 read time magzine and 42 read fortune, 20 read both NW and time, 25 read both NW and fortune, 15 read both time and fortune and 8 read all the three magzine

(i) find the no of people who read atleast one of the three magzine

(ii) fill in the correct no of people in each of the 8 regions of venn diagram.

(iii) find the no of people who read exactly one magzine.



Formula : $n(A \cup B \cup C) = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

total = 120

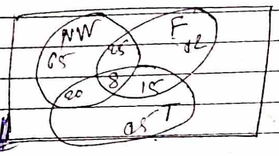
$n(NW) = 65$, $n(T) = 45$

$n(F) = 42$ $n(NW \cap F) = 20$

$n(NW \cap F) = 25$, $n(T \cap F) = 15$

$n(NW \cap F \cap T) = 8$

(i)



$n(NW \cup F \cup T) = n(NW) + n(T) + n(F) - n(NW \cap F) - n(NW \cap F) - n(T \cap F) + (NW \cap F \cap T) =$

$n(NW \cup F \cup T) = 65 + 45 + 42 - 25 - 20$
$15 + 8$

(ii) $65 + 45 + 50 - 25 - 20 - 15$

$160 - 60 = 100$

Q list the ele of each set where n
$n = \{1, 2, 3, 4, 5 - - - - n\}$

(i) ~~Set A such that = 00~~

$A = \{x : x \in N, 3 < x < 9\}$

(ii) $B = \{x : x \in N$ and x is even $x < 11\}$

(iii) $C = \{x : x \in N \quad 4 + x = 3\} \to$ null

(i) $\{4, 5, 6, 7, 8\}$

(ii) $\{2, 4, 6, 8, 10\}$

(iii) ϕ

* **Relations**: if we have two sets, set x and set y then the relation between them is represented using the ordered pair x, y such that $x \in X$, $y \in Y$

$A \times B = \{(a,b) \mid a \in A, b \in B\}$

$A \times B$, $R \subseteq A \times B$

$A = \{1, 2\}$, $B = \{3, 4\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

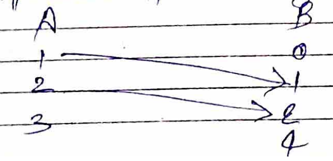
$R \subseteq A \times B$

$A R B$

Let set $A = \{1, 2, 3\}$
$B = \{0, 1, 4\}$

$A \times B = \{(1,0)(1,1)(1,4), (2,0)(2,1)(2,4)$
$(3,0)(3,1)(3,4)\}$

R is the relation where $A, B \in R$ if and only if $A = B$



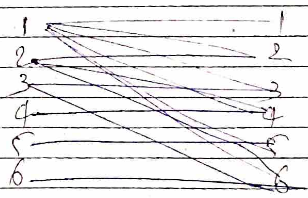
$A = \{1, 2, 3, 4, 5, 6\}$ Relation from a set to itself.

$A \times A = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)$
$(2,2)(2,3)(2,4)(2,5)(2,6)(3,1)(3,2)(3,3)(3,4)$
$(3,5)(3,6)(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)(5,1)$
$(5,2)(5,3)(5,4)(5,5)(5,6)(6,1)(6,2)(6,3)(6,4)$
$(6,5)(6,6)\}$

$\{a R b\}$
$\{a R b\}$

$R = \{(a,b) \mid a \text{ divides } b\}$ $b \in B\}$

$\{(1,1)(1,2)(1,3)(1,4)(1,6)(2,2)(2,4)(2,6)$
$(3,3)(3,6)(4,4)(5,5)(6,6)\}$



$A \times A = m \times n$
subset $= 2^{m}$

No of Relations on a set: $A \times A = m \times n$
Subset $= 2^{n^2}$

* $A = \{1, 2\}$ $B = \{a, b, c\}$

$A \times B$, $B \times A$, $A \times A$

$A \times B = \{(1,a)(1,b)(1,c)(2,a)(2,b)(2,c)\}$

$B \times A = \{(a,1)(b,1)(c,1)(a,2)(b,2)(c,2)\}$

$A \times A = \{(1,1)(1,2)(2,1)(2,2)\}$ | $(a,b) = (b,a)$ $A \times B = B \times A$

Set A and B = z^{+} ∈ integer

Consider the relation which contain

$R_1 = \{(1,1)(1,3)(2,1)(2,1)\}$

$R_1 = \{(a,b) \mid a = b\} \Rightarrow \{(1,1)\}$

$R_2 = \{(a,b) \mid a \leq b\} \Rightarrow \{(1,1)(1,3)(2,1)\}$

$R_3 = \{(a,b) \mid a > b\} \Rightarrow \{(2,1)\}$

$R_4 = \{(a,b) \mid a+b \leq 3\} \Rightarrow \{(1,1)(2,1)\}$

Domain and Range

Domain: Domain of a relation R is the set of all 1st elements of the ordered pairs which ∈ R

Range: Range of a relation R is the set of all 2nd element of the ordered pairs which ∈ R.

$A = \{1,2\}$ ~~$R = \{(1,a)(2,b)\}$~~

$B = \{a,b,c\}$

$A \times B = \{(1,a)(1,b)(1,c)(2,a)(2,b)(2,c)\}$

Domain Range

$R = \{(1,a)(2,b)\}$

$D = \{1,2\}$ $R = \{a,b\}$

Types of Relations / Properties of relations:

(i) Reflexive: A relation R on a set A is reflexive if aRa for every $a \in A$ that is if $(a,a) \in R$.

$A = \{1,2,3\}$

$R_1 = \{(1,1)(2,2)(3,3)\}$

$A = \{1,2,3,4\}$

$R = \{(1,1)(1,2)(2,2)(2,3)(3,3)(4,4)\}$

Reflexive

Irreflexive Relation: A Relation R on a set A is called irreflexive relation if $\forall a \in A$, $(a,a) \notin R$

$A = \{1,2,3,4\}$

$R_3 = \{(1,2)(2,1)(3,3)(3,4)\}$

irreflexive नहीं है because $(3,3) \in R$

$R = \{(1,2)(3,4)(2,1)(2,3)\}$

irreflexive है।

* Symmetric Relation : A Relation R on a set A is called symmetric if (b,a) ∈ R holds (a,b) ∈ R.

R = {(1,1) (1,2), (2,1) (2,2)}

R6 = {(1,1) (1,2) (1,3) (1,4)} Not Symm

* Anti-symmetric Relation : A Relation R on a set A is anti-symmetric if whenever (a,b) ∈ R and (b,a) ∈ R then (a = b).

(a,b) ∈ R, (b,a) ∈ R, (a = b)

Ex→ A = {1, 2, 3, 4}
R = {(1,1), (2,2) (3,3) (4,4)}

* Empty Relation : A Relation R on a set A is called Empty if the set A is an empty set that is any relation R where no element of set A is related to the element of set B.

Ex→ A = {1, 2, 3}   B = {5, 6, 8}
R = φ   No → R = {(1,5) (2,6)} (X)

* Transitive Relation : A Relation R on a set A is a transitive if whenever aRb and bRc then a should be related c (aRc)

a b c
A = {1, 2, 3, 4},   (a,b) ∈ R, (b,c) ∈ R, (a,c) ∈ R

R1 = {(1,1) (1,2) (2,3) (1,3) (1,4)} ✓
R2 = {(1,1) (1,2) (2,1) (2,2) (3,3) (4,4)} ✓
R3 = {(1,3) (1,1)} X
R4 = {φ} ✓ (Transitive)
R5 = A×A ✓

a = 1, b = 1
(a,b) (b,c)
(1,2)
(a,c) = (1,2)

* Inverse of a Relation : let R be any relation from a set A to set B the inverse of R is denoted by R^{-1} which is the relation from B to A that consist of those ordered pair which when reversed belong to R.

Ex→ A = {1, 2, 3}   B = {x, y, z}
R = {(1,y) (1,z) (3,y)}
R^{-1} = {(y,1) (z,1) (y,3)}

$\boxed{(R^{-1})^{-1} = R}$

* Compliment of Relation: Let R be any Relation from a set A to set B, the complement of Relation (R^c) is the relation from the A to B which consists of those ordered pair which $\notin R$.

$$\boxed{R^c = \{(a,b) \mid (a,b) \notin R\}}$$

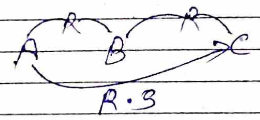
$A = \{1, 2\}$, $B = \{a, b, c\}$

$R = \{(1,a)(2,b)(2,c)\}$

$A \times B = \{(1,a)(1,b)(1,c)(2,a)(2,b)(2,c)\}$

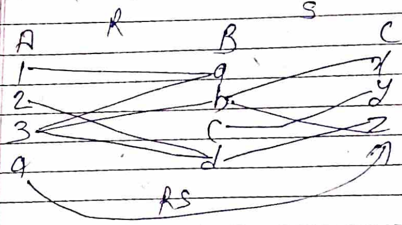
$R^c = \{(1,b)(1,c)(2,a)\}$

* Composition of Relation: let A, B, C be the three set let S be a relation from B to C that is R is Relation from A to B then R and S give rise to relation from A to C.



$A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$, $C = \{x, y, z\}$

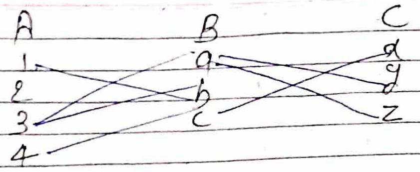
$R = \{(1,a)(2,d)(3,a)(3,b)(3,d)\}$
$S = \{(b,x)(b,z)(c,y)(d,z)\}$



$RoS = \{(2,z)(3,x)(3,z)\}$

Q. Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$
$C = \{x, y, z\}$ consider the relations are from A to B and S be the relation B to C.

$R = \{(1,b)(3,a)(3,b)(4,c)\}$
$S = \{(a,y)(c,x)(a,z)\}$
(i) Draw the diagram of RoS
(ii) write R^{-1} inverse and composition of RoS as set of ordered pair

① 

$R \circ S = \{(3, y)(3, z)(4, x)\}$

~~$(R \circ S)^{-1} = \{(y,$~~

(ii) $R^{-1} = \{(b, 1)(a, 3)(b, 3)(c, 4)\}$

(iii) ~~Rel~~

* Equivalance Relation: Consider a non empty set S, a relation R on S is an equivalance Relation if R is reflexive, symmetric and transitive relation

(i) For every $a \in S$, aRa such that $a = a$

(ii) If aRb then bRa

(iii) If aRb and bRc then aRc

poset with Ex → imp

(1M)

* partial ordering Relation: A Relation are on set S is called a partial ordering or a partial order of S if R is reflexive, antisymmetric and transitive.

A set S together with a partially ordering is called partial ordering set. → set R or poset.

(i) For every belong to S. aRa such $a = a$

(ii) if aRb, bRa / $a = b$

(iii) aRb, bRc, then aRc | Set $S = \{1, 2, 3, 4\}$

$Ex \Rightarrow \{(1,1)(2,2)(3,3)(4,4)\}$

(2M)

* closures properties of Relations.

consider a relation R
$R = \{(a, a)(a, b)(b, c)(c, c)\}$ on a set A that $A = \{a, b, c\}$
find.

(i) Reflexive closure
(ii) symmetric closure
(iii) Transitive closure

(1) Reflexive closure: The reflexive closure on R is obtained by adding all the diagonal pair of AXA to R which are ~~not~~ not currently there in R.

$R^+ = R \cup (b,b)$

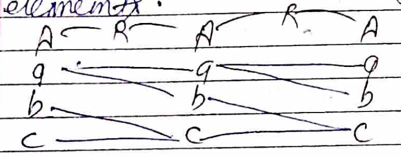
$R^+ = \{(a,a)(a,b)(b,b)(b,c)(c,c)\}$

(2) Symmetric closure: The symmetric closure on R is obtained by adding all the pairs in R^{-1} to R which are not currently in R.

$R^+ = R \cup \{(b,a)(c,b)\}$

$R^+ = \{(a,a)(b,a)(a,b)(b,c)(c,b)(c,c)\}$

(3) Transitive closure: The transitive closure on R, since A has 3 elements. $(R \cup R^2 \cup R^3)$



$\Rightarrow R \cup R.R.R$

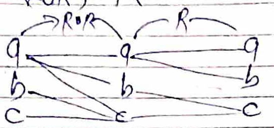
$(R \circ R) \circ R$

$R^2 = R \circ R = \{(a,a)(a,c)(b,c)(a,b)(c,c)$

```
public class A
{ public static int strMethod(String s)
  {
    return s;
  }
  public static ~~void~~ int add(int x, int y)
  { return x+y;
  }
  psvm(){
    strMethod("Raushan");
    add(5,5);
  }
}
```

$R \cup R^2 \cup R^3$

$R^3 = (R \circ R) \cup R$



$R^3 = \{(a,a)(a,c)(b,c)(c,c)(a,b)\}$

$\underbrace{R \cup R^2}\cup R^3$

ans $= \{(a,a)(a,b)(b,c)(c,c)\} \cup \{(a,a)(a,c)(b,c)(c,c)\} \cup \{(a,a)(a,c)(b,c)(c,c)(a,b)\}$

ans $= \{(a,a)(a,b)(a,c)(b,c)(c,c)\}$

$\boxed{R \cup R^2 \cup R^3 = \{(a,a)(a,b)(a,c)(b,c)(c,c)\}}$

* Transitive closure: A relation R is transitive if whenever $(a,b) \in R$ and $(b,c) \in R$ it follows that $(a,c) \in R$.

$A = \{1,2,3\}$

Ex → $R = \{(1,2)(2,3)\}$; $R = R \cup \{(1,3)\}$

→ $\{(1,2)(2,3)(1,3)\}$

~~$R^2 \circ R = R^3 = \{(a,a)(a,b)(a,c)(b,c)(c,c)\}$~~

Q1 considering the following 5 relation on the set A

$A = \{1,2,3\}$

1. $R = \{(1,1)(1,2)(1,3)(3,3)\}$
2. $R_2 = \{\phi\}$
3. $S = \{(1,1)(1,2)(2,1)(2,2)(3,3)\}$
4. $A \times A = \{(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)\}$
5. $T = \{(1,1)(1,2)(2,2)(2,3)\}$

Determine whether or not each of the above relation is a

(i) reflexive.
(ii) symmetric
(iii) transitive
(iv) anti symmetric

Q2 Let R and S be the following relation on $B = \{a,b,c,d\}$,

$R = \{(a,a)(a,c)(c,b)(c,d),(d,b)\}$

$S = \{(b,a)(c,c)(c,d)(d,a)\}$ Find the following Composition relation

(i) $R \circ S$
(ii) $S \circ R$
(iii) $R \circ R$
(iv) $S \circ S$

Q3 Let R be the relation on N defined by $x+3y=12$ that is

$R = \{(x,y) \mid x+3y=12\}$

R- (i) write R as set of ordered pairs
(ii) find the domain and range of R and R^{-1}

(iii) find the composition relation RoR.

(iv) find reflexive, symmetric transitive relation है की नहीं।

Q- The binary relation $R = \{(1,1)(2,1)(2,2)(2,3)(2,4)(3,1)(3,2)\}$ on the set $\{1,2,3\}$ you have to find all the relation.

Q4 simplify using property of set
$(x+y)(x+z)$

Sol : $x \cdot x + xz + yx + yz$ — identity or law $\{1+z=1\}$

~~$x + xz + yx + y \cdot z$~~

$x(1+z) + yx + y \cdot z$

$x + yx + yz$

$x(1+x) + yz$

$\boxed{\text{ans} = x + yz}$

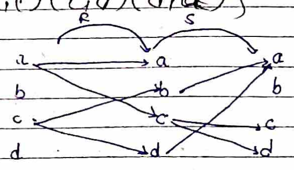
Q5 show that the relation are $R = \{(a,a)(a,b)(b,a)(b,b)(c,c)\}$ on a set A ~~show that it~~ is an equivalance relation also check whether it is partial or not.

Q6 If R is the Relation on the set $A = \{1,2,3,4\}$ defined by xRy if x exactly divides y. prove that (A,R) is poset (partial order set).

Q.7 Gives an example of a Relation which is symmetric but neither reflexive, not antisymmetric nor transitive.

Q8 Let R and S be relation on a set B where $B = \{a,b,c,d\}$, $R = \{(a,a)(a,c)(c,b)(c,d)(d,b)\}$
$S = \{(b,a)(c,c)(c,d)(d,a)\}$

(I) ROS
(ii) SOR



ROS $= \{(a,c)\ (c,a)\ (a,d)\}$.

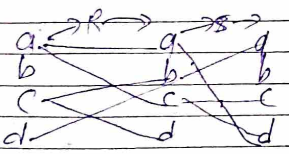
Q1 ans → Reflexive (i) NO (v) NO
(ii) NO
(iii) Yes
(iv) Yes

Symmetric → (i) NO (v) NO
(ii) NO
(iii) Yes

transitive → (i) NO (iv) Yes
(ii) NO (v) NO
(iii) Yes

Antisymmetry → (i) NO (iv) Yes
(ii) NO (v) NO
(iii) Yes

Q2. $B = \{a, b, c, d\}$
$R = \{(a,a)(a,c)(c,b)(c,d)(d,b)\}$
$S = \{(b,a)(c,c)(c,d)(d,a)\}$

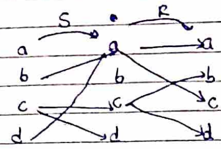
RoS →


$RoS = \{(a,d)(a,c)(c,a)(d,a)\}$

So on SoR, FOR SoS

Q3.
$R = \{(0,0)(6,2)(9,1)(12,0)(3,3)(0,12)\}$

Domain $D = \{0, 6, 9, 12, 3\}$
Range $= \{4, 2, 1, 0, 3, 12\}$
$R^{-1} = \{(4,0)(2,6)(1,9)(0,12)(3,3)(12,0)\}$

Q) B Ans


$S \circ R = \{(b,a)\ (c,b)\ (c,d)\ (d,a)\ (d,c), (b,c)\}$

$R = \{(a,a)\ (a,b)\ (b,a)\ (b,b)\ (c,c)\}$

Function

$R = \{(a,b)\}$

A → B
a — 1
b — 2
~~c — 3~~

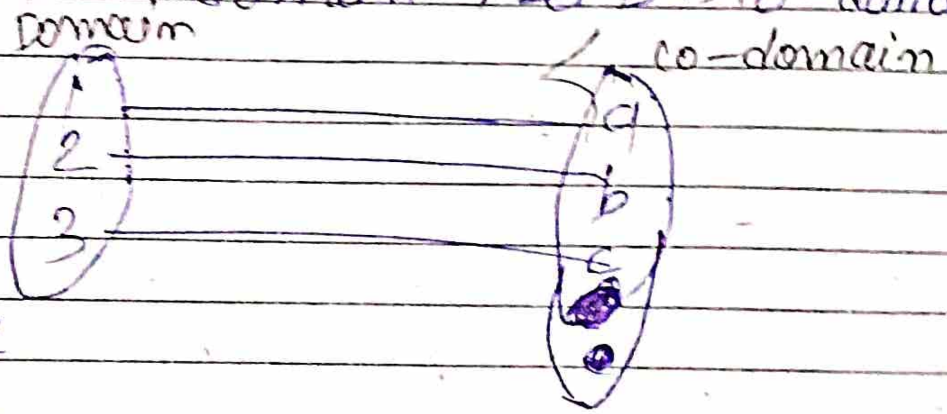
(2M) Define function? Also name the diff types of fun

function : A Relation f from a set A to a set B is called a function if, to each element of set A $a \in A$, we can assign unique element of set 'B'.

$A = \{1, 2, 3\}$ $B = \{a, b, c\}$

$f : A \to B$ $A \times B = \{(1,a)(1,b)(1,c)(2,a)(2,b)...$

Set A → domain, Set B → co-domain



f

$(x) \to f(x)$

A function assigns exactly one element of one set to each element of other sets.

A function is a rule that assigns each input exactly one output

A	B
a	1
b	2
c	3
d	

Set A = 4

Set B = 3

$\boxed{\text{ordered pair } A \times B = 12}$

$\boxed{\text{No of Relation: } 2^{12}} = 2^{ab}$

No of functions $= b^{a} = 3^{4}$

$\boxed{\text{Relation but not fun} = 2^{ab} - b^{a}}$

Set A = A = x $\quad A \times A = x^2$

No of R = 2^{x^2}

No of fun = x^{x}

total relat but not fun =

(No of Relation) − (No of fun)

$(2^{ab} - b^{a})$

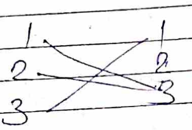
Q) Set A = {1, 2, 3} function

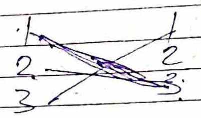
f = {(1,3)(2,3)(3,1)}

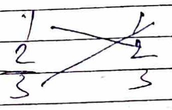
g = {(1,2)(2,1)}

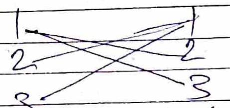
1) H = {(1,3)(2,1)(1,2)(3,1)}

which is function or not

f →  Yes fun

2) f =  Yes

(ii) g =  NO

3) H =  NO

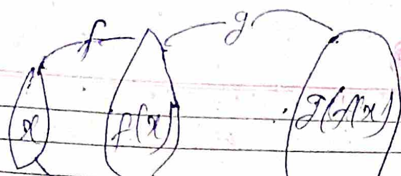
Composition of function:

Composite function: ~~let f is the function~~ let f and g are two functions such that f : A → B

g : B → C then composition of f and g denoted by gof is defined as the function gof : A → C such that

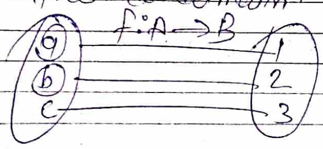
gof(x) = g(f(x))

EX ⇒ 4 page

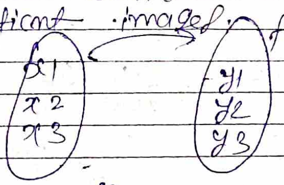


Types of function

① One to one (injective function) function:
A function which one ele. of the domain is connected to one element of the co-domain.



Different element should have different images.



$f(x) = y$

$x = y$

$f(x) = f(y)$

$x <= y$

if $x \neq y$

$f(x) \neq f(y)$

Q Suppose you have a function $f: R \to R$, $f(x) = ax + b$ $\forall x \in R$. Check whether the fn is one-one or not.

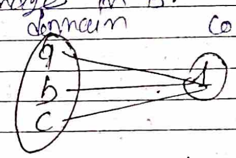
$f(x) = f(y)$

$ax + b = ay + b$

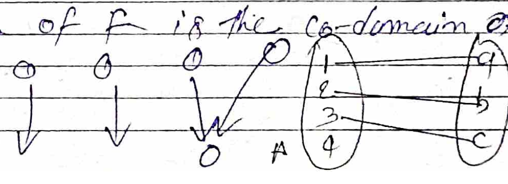
$\boxed{x = y}$

A function $f: A \to B$ is said to be injective fun if different different ele of A have diff images of B.

(ii) Many to one function: A function f such that $A \to B$ is said to be a many one function if two or more elements of set a have the same image in B.



③ Onto function / surjective: A function f such that $f: A \to B$, if every element of set B is an image of some element of A. ; $f(A) = B$
or
Range of f is the co-domain of f.



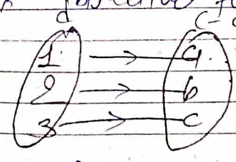
Q* check wheather following function are surjective or not

$g : \{1, 2, 3\} \to \{a, b, c\}$ defined by

$g = \left\{ \frac{1}{c}, \frac{2}{a}, \frac{3}{a} \right\}$

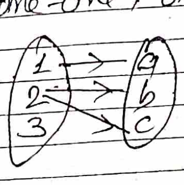
No surjective because b [illegible]

* Bijective function: It is known of one-one corrospondad function bijective function or one-one onto function, therefore if a function is both one-one and onto them. Such a function is bijective function.

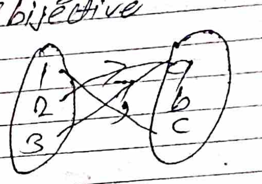


Q check wheather the following are one-one, onto or bijective

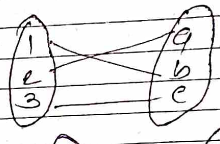
(i)



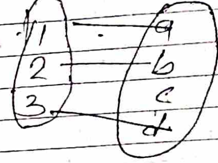
(ii)



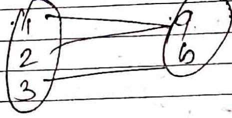
(iii)



(iv)



(v)



(i) Not a function

(ii) Neither one-one nor onto

(iii) one-one, onto ⇒ bijective

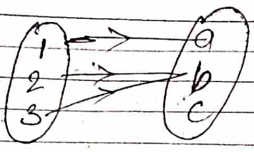
(iv) one-one

(v) onto

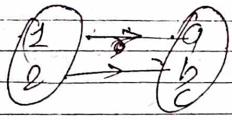
* Into function: A function f such that $f: A \to B$ A maps B said to be an into function

with ~~which~~ if there exist an ele. in B no image in set A.

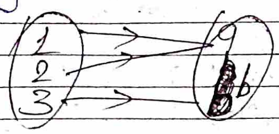
A function is into function when it is not onto



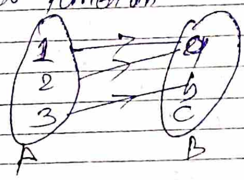
* one-one Into function : A function which is both one-one and into called one-one into function.



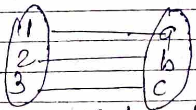
* Many-one onto function : A function which is both many-one and onto is called many-one ~~an~~ onto function.



* Many-one Into function : A function many-one and Into function is called Many-one-into function.



* Inverse function :



$x = f(a)$ | $f(x) = x^2$

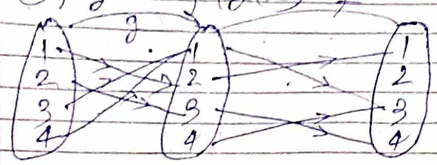
$a = f^{-1}(x)$ | $f^{-1}(x^2) = x$

4 Marks

Q) Let $V = \{1, 2, 3, 4\}$ you have two functions. $f: V \to V$ $g: V \to V$ find $f(g)$, $g(f)$. where $f = \{(1,3)(2,1)(3,4)(4,3)\}$, $g = \{(1,2)(2,3)(3,1)(4,1)\}$

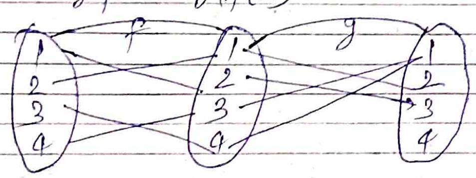
Example of Composition of function

ans.> (1) $fog = f(g(x))$



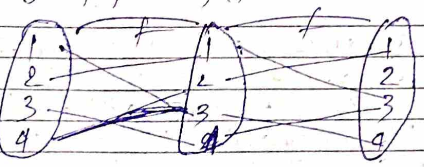
$fog = \{(1,1)(2,4)(3,3)(4,3)\}$

(2) $gof = g(f(x))$



$gof = \{(1,1)(2,2)(3,1)(4,1)\}$

(3) $fof = f(f(x))$



$fof = \{(1,4)(2,3)(3,3)(4,4)\}$

Q Let f is function such that it maps $R \to R$ similarly
$g : R \to R$
$f : R \to R$ defined by $f(x) = 2x+1$
$g(x) = x^2 - 2$ find out gof ?

$gof = g(f(x))$

$= x^2 - 2 \quad (x = 2x+1)$

$= (2x+1)^2 - 2$

$= 4x^2 + 1 + 4x - 2$

$\boxed{gof = 4x^2 + 4x - 1}$

$fog = f(g(x))$

$= 2x+1 \quad x = x^2 - 2$

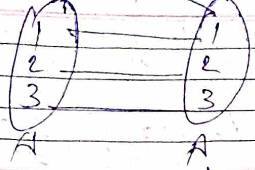
$= 2(x^2-2)+1$

$= 2x^2 - 4 + 1$

$\boxed{fog = 2x^2 - 3}$

Q find inverse of a function $f : A \to A$
$A = \{1, 2, 3\}$

Ans



$f^{-1} = \{1, 2, 3\}$

* Tute (2 marks)

Q1. Let $A = \{1, 2, 3, 4\}$ define a ~~Real~~ Relation on ~~Set~~ A which is reflexive and transitive but not symmetric.

Q2 For any set $x = \{1, 2, 3, 4, 5\}$ write all the proper and improper subset of set x.

Q3. Give an example of a function which is onto but one-one.

Q4. prove demorgan's law.

Q5 Relation R on N defined by $x+3y=12$, and $R = \{(x,y) : x+3y = 12\}$

(i) write the ordered pairs of R.

(ii) find domain and Range of R.

(iii) find R^{-1}.

(iv) find RoR.

(V) Set $A = \{a, b, c\}$, $B = \{x, y, z\}$ set $C = \{r, s, t\}$ g is a relation to $A \to C$ (A to C) $g = \{x, s, y, t, z, r\}$.
$F = \{a, y, b, x, c, y\}$

(i) gof (ii) fog

Q6. which function is serjective

f = id mapp $Z \to Z$ defined by

(1) $f(n) = 3x$

(2) $h : \{1, 2, 3\} \to \{1, 2\}$ defined

Q7 determine whether the following relation is reflexive, symmetric, antisymmetric or transitive on set $S = \{1, 2, 3\}$

$R = \{(1,1)(1,2)(2,1)(2,2)(3,3)\}$

MST

Q8 prove that $(A \cup B)^c = A^c \cap B^c$

Q9 use venn diagram to show

(1) $A^c - \cap (B - C^c)$

(2) $A^c \cup (B \cap C)$

Q10 let R be a relation, $R = \{(1,1)(1,2)(2,3)\}$ on a set $A = \{1, 2, 3\}$ find reflexive and symmetric of R.

Q11 In a servey of 120 people 65 like T, 20 both T & C, 45 like C, 42 like B, 15 like both C & B, 8 like All (i) find No of people who like at least one of the three

(ii) find no of people who like only one.

Q Give an example of Relation which is symmetric and transitive but not reflexive (ii) irreflexive and transitive

Q In a servey of 25 students it was found that 18 took maths, 12 took P, 13 took C, ~~[illegible]~~ 11 took math and C, 8 took M & P, 4 took P & C, 3 took all three sub.

(i) find the students that had taken only one of this subjects

(ii) find the no of students that had taken none of the subjects.

Q Three function F, g, h

$f, g, h : R \to R$

↓ Real no

R defined by $f(x) = x+2$, $g(x) = \frac{1}{x+1}$, $h(x) = 3$ (i) find out $g(h(f))$ (ii) $f(g(f))$ (iii) $f(g(h))$

Q Let R and S be the following relation on A Set $A = \{1, 2, 3\}$

$R = \{(1,1)(1,2)(2,3)(3,1)(3,3)\}$

$S = \{(1,2)(1,3)(2,1)(3,3)\}$ find out

$R \cup S$

R^{1}

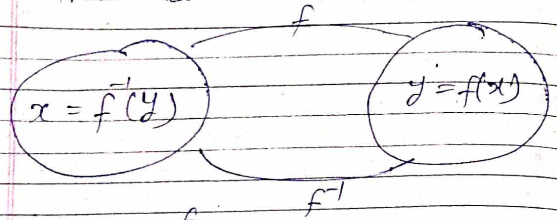
$R \circ S$

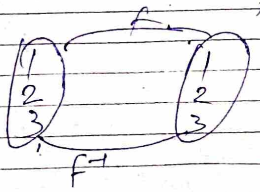
S^{2}

$R - S$

R symmetric diff

Inverse function: If f is a funⁿ $f: X \to Y$ is a bijection then there always exists a pre-image $f^{-1}(y)$ $\forall$ ele y of the set Y this will be a unique ele of X.





Ex → f is a function to $R \to R$ such that $f(x) = 2x-3$

$y = f(x)$

$f^{-1}(y) = x$

$y = 2x-3$

$x = ?,\ y = 3$

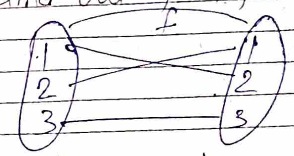
$y = 2x-3$

$y+3 = 2x$

$\boxed{x = \frac{y+3}{2}}$

Q Let $A = \{1,2,3\}$ define a function $f: A \to A$ given $f = \{(1,2)(2,1)(3,3)\}$ find out f^{-1}, f^2, f^3.

Solⁿ



$f^2 = fof$

$f^3 = fofof = \underbrace{fof}_{f^2}$

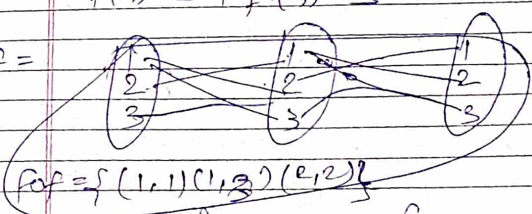
$f^3 = fof^2$

$f(1) = 2$ | $f^{-1}(2) = 1$

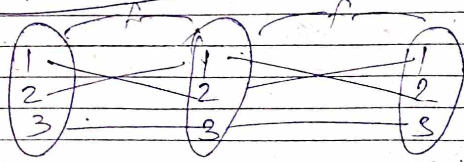
$f(2) = 1$ | $f^{-1}(1) = 2$

$f(3) = 3$ | $f^{-1}(3) = 3$

$fof =$

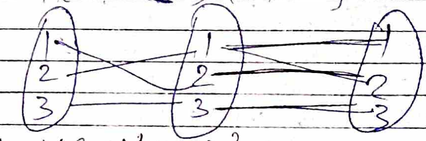


$fof = \{(1,1)(1,3)(2,2)\}$



$f^2 = \{(1,1)(2,2)(3,3)\}$

$f^3 =$



$f^3 = \{(1,2)(2,1)(3,3)\}$

Q. $f(x) = x^2$, $g(x) = x+3$

$g \to g \to g$ show that $fog \neq gof$

$fog = f(g(x))$

$fog = f(x+3)$

$= (x+3)^2$

$gof = g(f(x))$

$= g(x^2)$

$= x^2+3$

$$\boxed{(x+3)^2 \neq x^2+3}$$

Hashing : In the process of hashing we are able to convert the large data items into a smaller table, we use hash function to map the data.

Range of key values can be converted into a range of indexes of an array with the help of hashing. hash function is used or applied to a key, this key is used to generate an integer and this integer is used as an address in the hash table.

methods of hashing

(i) division method
(ii) Mid square method
(iii) folding

(i) Division method :

$h(k) = k \bmod m$

h → hash function, k → key, m → table size $(0 \leftrightarrow m-1)$

Q1. Key = 10, 13, 74, 21, 5, 19, 27

Size of hash table = 10

$m = 10$

⇒ 10 mod 10 = 0,

13 % 10

74 % 10

21 % 10

5 % 10

0	10
1	21
2	
3	13
4	74
5	5
6	
7	27
8	
9	19

Q. Assume $h(k) = K \% 111$, the record of the customers will be assigned by the hash function with the social security number of keys to memory locations.

keys = 064212848, 037149212, 107405723

ans → (i) $064212848 \% 111 = 14$

(ii) $037149212 \% 111 = 65$

(iii) $107405723 \% 111 = 14$

0	
14	064212848
15	037149212
65	037149212

* Recursively defined function: A function is called itself again and again is called Recursively defined function.

* Basic counting principle: The fundament counting principle is a way of finding how many possibilities can exist when combining choices. It is some times called ~~choices. It is so~~ fundamental counting rule or the product rule or the multiplication and the rule of multiplication.

If an event A can occure in n ways following which another event be (independent of A) can occure in m different ways., then the total occurence of both the events. A and B in given order is $m \times n$.

In general if there are n events occuring independently then all events can occure in the order indicated as $n_1 \times n_2 \times n_3$ ...... n ways.

Ex → A(3)

Chandigarh — Delhi

· Train — Auto

· Bus — Cab

· Air

so using multiplication rule
$m \times n = 3 \times 2 = 6$

+ Sum rule principle: Assume some event E can occure in m ways and a second event F can occure in n ways.

E → m and both the event
F → n canot occure simultaneously
then E or F can occure in
m+n ways.

In general there are n events and no to event can occure at the same time then the event can occure
$m_1 + m_2 + m_3 + m_4$ ———— n ways.

Q. write the solution set of the equation $x^2 - 4 = 0$ in roster form

Q2. Let $A = \{a, b, c, d\}$, set $B = \{1, 2, 3\}$ define the functions f & g as follows $f: A \to B$ defined by $f(a) = 2$, $f(b) = 3$, $f(c) = 1$, $f(d) = 2$ and g is the function $g: A \to B$ defined by $g(1) = 3$, $g(2) = 1$, $g(3) = 2$

Q. Create arrow diagrams for the function f & g and calculate fog, gof, fof, gog.

Q3 Set $A = \{1, 4, 9, 16, 25 ----\}$ write in set builder form.

Q4. If $A = \{3, 5, 7, 9, 11\}$
$B = \{7, 9, 11, 13\}$
$C = \{11, 13, 15\}$
calculate $A \cap (B \cup C)$

Q4. List the elements in each of the following sets let $U = \{0, --- 10\}$
$A = \{0, 1, 2, 3, 5, 8\}$ $B = \{0, 2, 4, 6\}$
Set $C = \{1, 3, 5, 7\}$ calculate $A \cup B$, C
B^c, $A \cap B^c$, $A^c \cup C$, $(A^c \cap C) \cup B$, $(A \cup B)^c$
$(A \cup C) \cap B$, sub set of A.

Q5 Let f is the function $R \to R$ f; g: $R \to R$ defined by
$f(x) = 2x + 1$, $g(x) = x^2 - 2$
find: fog, gof, gof, fof

Q6. give an example of a function which is on to a but not one - one.

unit

Combinatorial Mathematics

: If an event A can occure in m different way following which another event B (independent of 'A) can occure in n different ways, then the total occurence of both the events A and B is given by $m \times n$.

* Sum Rule principle: If some event E can occure in m way and a second event F can occure in n different ways and suppose both the events can not occure simultaneously them E or F can occure in 'm+n' ways

Q In a bag bob has 5 purple balls 4 green balls and 6 red balls. If ball has to draw a ball at random from the bag what is the probability.

Q2 If bob wants to take a trip to the batch bob can travell to 37 internationals batches or on of the 140 domestic batches how many choices does bob have for a batch vaction.

Q3 Suppose a college have 3 different history courses 4 diff literature courses and 2 diff psycology.

(i) The no n of ways a student can choose one of each kind of courses ⇒ $3 \times 4 \times 2 = 24$

(ii) The number n of ways a student can choose just one of the courses

ans ⇒ $3+4+2 = 9$

trick ⇒ अगर एक ही लेना है और चार चीज दिया हुआ है तो add करना है।

Ex ⇒ 4 red, 5 g, 6 p ~~में~~ से pen लेना है। = (i) No of ways

ans ⇒ $4+5+6 = 15$ pen

(ii) हर एक में से ~~एक~~ एक लेना है,

$4 \times 5 \times 6 = 120$

\# factorial : Multiplication of n natural no i.e $n!$

$$\frac{1}{6!} + \frac{1}{8!} + \frac{1}{10!} =$$

$$\frac{1}{6!}\left(1 + \frac{1}{8\times7} + \frac{1}{10\times9\times8\times7}\right)$$

$$\frac{1}{6!}\left(1 + \frac{1}{56} + \frac{1}{5040}\right)$$

$$\frac{1}{6!}\left(\frac{5040 + 90 + 1}{5040}\right)$$

$$\frac{1}{6!}\left(\frac{5131}{5040}\right)$$

Q: $6!(1\cdot3\cdot5 \ldots 11)\cdot2^6$

Multiply & divide by $2\times4\times6\times8\times10\times12$

$$\frac{6!\times12!\times2^6}{2\times4\times6\times8\times10\times12} = \frac{6!\times12!\times2^6}{2^6\times6\times10\times12}$$

$$= \frac{6\times5\times4!\times12\times11\times10\times9!}{6\times10\times12}$$

99
56
594
495
559

12
72

35574
72
11088
3880 8X

0
399168

$= 5 \times 4! \times 11 \times 9!$

$= \cancel{120} \times 12 \times 10 \times 11 \times 9!$

$12 \times 11 \times 10 \times 9! = 12!$

Q How many odd nos can be formed by using the digit 0, 2, 5, 7 when repetation is not allowed

How many 4 digit nos greater than 1000 can be formed with digits 0, 1, 2, 3, 4 ~~repetation~~
(i) repeation allowed
(ii) repeation not allowed

Q3. There are 4 rutes b/w delhi to goa in how many diff way can a man go from delhi to goa and return if for returing
(i) any of the root can be taken
(ii) Same rute can not taken

Sol. 0, 2, 5, 7.

5 — 1
7 — 1
57 — 1
75 — 1
25 — 1
52 — 1
27 — 1
72 — 1

257 — 1
275
725 — 1
752 — 1
527 — 1
205
207
275 = 24.
4!

Sol. 1 × 4 × 3 × 2 = 24 = 3.

4 × 5 5 5 = 125 − 1 = 124

Tutorial:

Q1 Define and give Example:

(i) Partial order relation
(ii) Equivalence relation
(iii) Total order relation;
(iv) Hashing function
(v) Inverse relation
(vi) Integral Domain

Q2. A class consist of 40 girls & 60 boys, in how many ways can be a President, vice president, ~~treas~~ treasurer & Sectary be choosen, If treasurer must be a girl, the Sectary must be a boy & a student may not hold more than 1 offers.

Q3. In how many ways 6 maths books, 5 eng books, can be arranged on a book shelf, Also find no of ways
(ii) all Eng books kept together either at the start of the shelf or at the end of the shelf

~~Q (ii) All english books kept together either at the~~

(i) Eng books should be kept together always

Q4 In how many ways can the letters of the word can be arranged.

(1) LEADER (2) ACADEMY.

formula $= \dfrac{\text{total word!}}{\text{repeated w!}}$ $\Rightarrow$ (1) $\Rightarrow \dfrac{6!}{2!} \Rightarrow \dfrac{6\times5\times4\times3\times\cancel{2!}}{\cancel{2!}} = 360$

(ii) $\dfrac{7!}{2!} = \dfrac{7\times6\times5\times4\times3\times\cancel{2!}}{\cancel{2!}} = 360\times7 = 2520$

Q5 Find the no of words that can be formed with the letters of the word
(i) "UNIVERSAL" such that the vowels remain together always
(ii) NATION

(i) $\dfrac{[\text{group vowels}]\text{ then count word}! \times \text{total}!}{\text{repeated word}!}$

(i) $[UIEA] \Rightarrow 4 \Rightarrow \dfrac{6!\times4!}{0!} = 6\times5\times4\times3\times2\times1\times4\times3\times2\times1 = 17280$

(ii) Nation $\Rightarrow$ N(AIO) $\Rightarrow 3 \Rightarrow \dfrac{4!\times3!}{2!} =$

$\dfrac{4\times3\times\cancel{2!}\times3!}{\cancel{2!}} \Rightarrow 4\times3\times3\times2\times1 = 72$

permutation | Combination
Arrange | Select

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}} \qquad \boxed{{}^nC_r = \frac{n!}{r!(n-r)!}}$$

* Permutation : Each of the arrangement which can be made by taking some, or all of a number of things is called permutation.

$1 \le r \le n$ denote ⇒ ${}^nP_r$ or $P(n,r)$

n → total no of observations
r → random no of observations that are required.

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

Q In how many ways 3 diff rings can be worn in 4 fingers with at most one in each finger.

$n = 4$
$r = 3$

$$\frac{4!}{(4-3)!} = 4 \times 3 \times 2 \times 1 = 24 \text{ ways}$$

* If there are three objects then the permutation of these objects taking two at a time is ?

$n = 3 \quad r = 2$

$${}^nP_r = {}^3P_2 = \frac{3!}{1!} = \frac{3 \times 2 \times 1}{1} = 6$$

Note: (i) If the elements occure always together,
Permutation of n diff thing taken all at a time in which P things always occure together, then permutation. $= P!\,(n-P+1)!$

(ii) If question Always included
Permutation of n diff things taken all at a time in which x perticular thing are always included.

$$\boxed{{}^rP_x \times {}^{n-x}P_{r-x}}$$

(iii) Always Excluded : Permutation of n diff things taken all at a time in which x perticular things are always excludeed.

$n-x$
p
x

Q. Let's suppose I have "Leagond" make 5 letter word.

$n = 6$ permutation.
$r = 5$

$$\frac{6!}{(6-5)!} = \frac{6\times5\times4\times3\times2}{1}$$

$= 30 \times 24 = 720$

Q. A same question but

Q. A commettee of 5 persons is to be formed from 6 men and 4 women, in how many ways can this be done with atleast two women should be included.

Combination because we are selecting

$6C_3$ 3M	2W	$4C_2$
$6C_2$ 2M	3W	$4C_3$
$6C_1$ 1M	4W	$4C_4$

5 P — 2W

$n=6$, $r=3$: $6C_3$	3M	2W	$4C_2$
$6C_2$	2M	3W	$4C_3$
$6C_1$	1M	4W	$4C_4$

$6C_3 \times 4C_2 + 6C_2 \times 4C_3 + 6C_1 \times 4C_4$

$= \frac{6!}{3!\,3!} \times \frac{4!}{2!\,2!} +$

Q. A box contains 4 red 3 white and 2 blue balls, 3 balls are drawn at random find the no of ways of selecting the balls of diff colors.

4R, 3W, 2B

$4C_1 \times 3C_1 \times 2C_1$

$\frac{4\times3!}{1!\,(3)!} \times \frac{3\times2!}{1!\,(2)!} \times \frac{2\times1!}{1!\,(1)!} = 4\times3\times2 = 24$ are

$\frac{1}{3\times2} \times \frac{1}{24} \times \frac{1}{7} = \frac{1}{15}$

Recurrence

~~Reference~~ Relation: A recurrance relation is an equation that recursively defines where the next term is a function of the previous term. It represents a sequence based on some rule.

* A series or a sequence generated by a recurrence relation is called a recurrence sequence

* Methods to solve reccurence relation

1. Back substitution
2. Master theorem
3. Recursion tree.

$S_n = \{1, 5, 9, 13, \ldots\}$

$S_n = S_{n-1} + 4$ (RR)

$S_n = \{2^1, 2^2, 2^3 \ldots\}$

$S_n = \{2^n\}$

$\boxed{RR = S_n = 2(S_{n-1})}$

$S_n = \{1, 1, 2, 3, 5, 8, 13 \ldots\}$

$\boxed{\begin{array}{l} S_1 = 1 \\ S_2 = 1 \end{array}}$ $S_n = (S_{n-1}) + (S_{n-2})$

$n \geq 1$

* Linear reccurence relation with constant coefficient.

$a_n = a_{n-1} + a_{n+2} + \ldots a_{n-k}$

coefficient →

LRR → $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \ldots + c_k a_{n-k}$,

(1) $2a_n + 3a_{n-1} = 3$

(2) $a_n - 12a_{n-1} + 6a_{n-2} = n \cdot 5^n$

(3) $a_n = a_{n-1} + a_{n-2}$

(4) $a_n = a_{n-1} \cdot a_{n-2}$

(5) $a_n \cdot a_{n-1} = a^2_{n-2}$

Determine linear reccurence relation

(1) Yes → 2 Yes → 3 Yes → 4 Yes → 5 NO

Degree of linear recc rela = 1

$a_n = a_{n-1} + a_{n-2}$

order $= n - (n-2) = 2$

* Methods to solve linear recurrence relation.

(i) Iterative method
(ii) Method of characteristic ~~rates~~ roots
(iii) generating function.

* Types of reccurences with constant coefficient

(i) Linear Homogeneous reccurance relation

$a_n = c_1 \cdot a_{n-1} + c_2 a_{n-2} + \ldots c_k \cdot a_{n-k}$,
where $\boxed{a_n = 0}$

(ii) Non-Linear Reccurance relation
(iii) linear non homogeneous reccurence relation with constant coefficient

* Steps to solve Linear Homogeneous reccurence relation.

(i) $a_n = r^n, a_{n-1} = r^{n-1}, a_{n-2} = r^{n-2} \cdots$

(ii) find an equation in terms in r this is called of

(iii) characteristic equation or auxulary equat^n.

(iv) solve the chara equation and find chara roots.

(v) If the characteristic roots are

(I) $r = r_1, r_2, r_3$. then general solution is:

$$\boxed{a_n = b_1 r_1^n + b_2 r_2^n + b_3 r_3^n +}$$

(ii) if $r = r_1, r_1$ then the general sol^n is:

$$\boxed{a_n = (b_1 + n b_2) r_1^n}$$

$r = r_1, r_1, r_1$

$a_n = (b_1 + n b_2 + n^2 b_3) r_1^n$

(iii) If $r = r_1, r_1, r_2, r_3$

$a_n = (b_1 + n b_2) r_1^n + b_3 r_2^n + b_4 r_3^n$

(iv) If $r = r_1, r_1, r_1, r_2$

$a_n = (b_1 + n b_2 + n^2 b_3) r_1^n + b_4 r_2^n$

Q① $4a_r + 12a_{n-1} = 3$ find root

Q② $6a_n - 24a_{n-1} - 16a_{n-2} = 0$

Q③ $a_n - a_{n-1} + a_{n-2} - a_{n-3} = 0$

Q④ $a_n = 2a_{n-1} + 1 \Rightarrow a_n - (2a_{n-1}) = 1$

① order = Highest subscript - lower subscript

$\Rightarrow n - (n-1) = n - n + 1 = 1$

Q solve

$a_n = a_{n-1} + 2a_{n-2}, n \geq 2$ with the initial condition

$a_0 = 0, a_1 = 1$

Solⁿ:

$a_n = a_{n-1} + 2a_{n-2}$

$a_n - a_{n-1} - 2a_{n-2} = 0$

It is homogeneous

Step 1: Put $a_n = r^n, a_{n-1} = r^{n-1}, a_{n-2} = r^{n-2}$

$r^n - r^{n-1} - 2r^{n-2} = 0$

$r^n - r^{n-1} - 2r^{n-2} = 0$

$r^n \left[1 - \frac{1}{r} - \frac{2}{r^2}\right] = 0$

$r^n \left[\frac{r^2 - r - 2}{r^2}\right] = 0$

$r^2 - r - 2 = 0$

$r^2 - 2r + r - 2 = 0$

$r(r-2) + 1(r-2) = 0$

$r - 2 = 0, \; r + 1 = 0$

$\boxed{r = -1, 2}$ $\boxed{r = 2, -1}$

root are distinct so general solution

is: $a_n = b_1(2)^n + b_2(-1)^n$ $\boxed{a_n = b_1 r_1^n + b_2 r_2^n}$

$a_0 = b_1(2)^0 + b_2(-1)^0$

$0 = b_1 + b_2$ — ①

$a_1 = b_1(2)^1 + b_2(-1)^1$

$a_1 = 2b_1 - b_2$

$\boxed{1 = 2b_1 - b_2}$ — ②

Solve eqⁿ ① & ②

from eq^n (1)

$\boxed{b_1 = -b_2}$ put in eq 2

$1 = 2(-b_2) - b_2$

$1 = -2b_2 - b_2$

$1 = -3b_2$

$\boxed{b_2 = -\frac{1}{3}}$ $\boxed{b_1 = \frac{1}{3}}$

$\boxed{a_n = \frac{1}{3}(2)^n + \frac{1}{3}(-1)^n}$ solution

Q $a_n = 4(a_{n-1} - a_{n-2})$ with the initial condition $a_0 = 1,\ a_1 = 1$

Sol: $a^n = 4a_n - 4 - 4a_n - 8$

$a_n - 4a_n + 4 + 4a_n + 8 = 0$

It is homogeneous

Now, put $a_n = r^n$

$r^n - 4r + 4 + 4r + 8 = 0$

$r^n + 12 = 0$

$r^n = 12$

~~$a_n = 4a_{n-1}$.~~

$a_n = 4a_{n-1} - 4a_{n-2} \Rightarrow a_n - 4a_{n-1} + 4a_{n-2} = 0$

It is homogeneous

put $a_n = r^n$

$r^n - 4r^{n-1} + 4r^{n-2} = 0$

$r^n\left[1 - \frac{4}{r} + \frac{4}{r^2}\right] = 0$

$\frac{r^2 - 4r + 4}{r^2} = 0$

$r^2 - 4r + 4 = 0 \Rightarrow r^2 - 4r + 4 = 0$

$r^2 - 2r - 2r + 4 = 0$

$r(r-2) - 2(r-2) = 0$

$\boxed{r = 2, 2}$

Since the roots are equal then the general solution will be:

$a_n = (b_1 + n b_2) r_1^n$

$a_n = (b_1 + n b_2) 2^n$ initial conditions are $a_0 = 1, a_1 = 1$

$a_0 = (b_1 + 0 b_2) 2^0$

$1 = b_1 2^0$ $\boxed{1 = b_1}$ $a_1 = (b_1 + b_2) 2^1 \Rightarrow a_1 = 2b_1 + 2b_2$

$2b_1 + 2b_2 = 1 \Rightarrow 2b_2 = 1 - 2b_1$ $b_2 =$ $\boxed{a_n = \left(1 - \frac{1}{2}n\right) 2^n}$

MST - 1

(2m) Identify the smallest relation containing the relation {(1,2)(1,4)(3,3)(4,1)} defined on set A = {1,2,3} that is
(a) Reflexive
(b) Symmetric
(c) transitive

(a) steps to obtain the smallest relation:

(i) Reflexive : For a reflexive relation each element in set A must be related to itself i.e. (a,a) ∈ R ∀ a ∈ A.

The set A = { 1,2,3,4 }
then the reflexive closure
{(1,1)(2,2)(3,3)(4,4)}
Since (3,3) already present we don't need to add it again

Now, R becomes:

R = {(1,2)(1,4)(3,3)(4,1)(1,1)(2,2)(4,4)}

(2) Symmetric : A relation is symmetric if for every (a,b) ∈ R, (b,a) also ∈ R.
for :
(1,2) we need (2,1)
for (1,4) ;(4,1) is already there

we need to add (2,1)

R = {(1,2)(1,4)(3,3)(4,1)(1,1)(2,2)(4,4)(2,1)}

3. Transitive : A relation is transitive if, whenever (a,b) ∈ R and (b,c) ∈ R then (a,c) must be belong to R.
we need to check all pairs for transitivity.
- (1,2) and (2,1) (1,1) already there
- (1,4) (4,1) (1,1) already there

(4,1) and (1,2) so we need to add (4,2)

~~(4,2) and (1,2) so (4,2)~~ al.
(4,2) and (2,1) imply (4,1) already there

R = {(1,1)(1,2)(1,4)(2,1)(2,2)(3,3)(4,1)(4,2)
(4,4)}
Final answer : The smallest relation containing {(1,2)(1,4)(3,3)(4,1)}

R = {(1,1)(1,2)(1,4)(2,1)(2,2)(3,3)(4,1)(4,2)
(4,4)}

4)Q.- Given $f(x) = 3x^2 + 4x + 7$. find
$g(x) = x+1$
(a) fog (b) gof

(a) $fog \rightarrow$ This means we substitute $g(x)$ into $f(x)$.

$f(g(x)) \Rightarrow f(x+1) \Rightarrow 3(x+1)^2 + 4(x+1)+7$

$\Rightarrow 3(x^2+1+2x) + 4x+4+7$

$\Rightarrow 3(x^2+2x+1) + 4x+11$

$\Rightarrow 3x^2+6x+3+4x+11$

$\Rightarrow \boxed{3x^2+10x+14}$

Thus, $f(g(x)) = 3x^2+10x+14$

(b) $g(f(x)) \Rightarrow g(3x^2+4x+7)$

$\Rightarrow (3x^2+4x+7)+1$

$\Rightarrow 3x^2+4x+8$

Thus, $g(f(x)) = 3x^2+4x+8$

4M) Compute whether the following relations are equivalent or partial order relations

(i) $A = \{2,3,4\}$
$R = \{(2,2)(3,3)(4,4)(2,3)(3,4)\}$

(ii) $R = \{(x,y) : y = x+5 \text{ \& } x<4,\ y \in R\}$

(i) Relation R on set $A = \{2,3,4\}$
$R = \{(2,2)(3,3)(4,4)(2,3)(3,4)\}$

Now for equivalent relation any relation must be reflexive, transitive and symmetric.

(i) Reflexive : $\forall$ ele $a \in A$; $(a,a) \in R$

(ii) Symmetric : if $(a,b) \in R$ ~~and~~ then $(b,a) \in R$

(iii) Transitive : if $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$

(i) Reflexive : $(2,2)(3,3)(4,4) \in R$ so it is reflexive

(ii) Symmetric : $(2,3) \in R$ but $(3,2) \notin R$ so it is not symmetric

(iii) Transitive : $(2,3) \in R$ but $(3,4) \in R$ ~~...~~ but $(2,4) \notin R$ ~~...~~ so it is not transitive

since for equivalent relation, any relation must be reflexive, symmetric and transitive but there relation is neither symmetric nor transitive thus this is not ~~...~~ equivalent relation.

Partial order Relation: Since for partial ~~order~~ order relation, any relation must be reflexive, Antisymmetric and transitive

Since, we already check that relation is reflexive but not transitive thus this is also not partial order relation

(2) $R = \{(x,y) : y = x+5$ and $x < 4\}$

for $x = 0$; $y = 5$
$x = 1$; $y = 6$
$x = 2$, $y = 7$
$x = 3$, $y = 8$

Thus, $R = \{(0,5)(1,6)(2,7)(3,8)\}$

Now, let's check if this relation is a equivalent or partial order.
for equivalent relation, any relation must be reflexive, symmetric and transitive

(i) Reflexive : the relation does not contain pairs like (0,0) (1,1) (2,2) (3,3) so it is not reflexive

Since at first step relation is not reflexive that means relation neither equivalent nor partial order.

2m Q. Define partial order relation with example.

A Relation will partial order relation if given relation would be.
(i) reflexive relation
(ii) ~~Symmetric relation~~ Antisymmetric
(iii) transitive relation

(i) Reflexive : $\forall$ ele $a \in A$, $(a,a) \in R$

(ii) Antisymmetric : if $(a,b) \in R$ and $(b,a) \in R$, then $a=b$

(iii) Transitive :
if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$

Example : $A = \{1,2,3\}$
$R = \{(1,1)(2,2)(3,3)(1,2)(1,3)(2,3)\}$
the given example is a partial order because
(i) reflexive : (1,1)(2,2)(3,3) belong to R
(ii) Antisymmetry : There are no pairs of the form (a,b) and (b,a) where $a \neq b$ so, the relation is antisymmetric.
(iii) Transitive : ~~if (a,b) and~~ the relation satisfies transitivity because (1,2) and (2,3) imply (1,3) which is present in the relation
There are no other pairs that violate transitivity
Since the relation satisfies all three properties, it is a partial order.

Q2. In how many ways ~~three different~~ can the letters of the word "SPECIAL" be arranged in a row such that the vowels occupy only odd positions.

SPECIAL = 7

vowel = EIA Consonants = SPCL

1 2 3 4 5 6 7

odd odd odd odd

4 odd places and we have to arrange 3 vowels at odd posn.

$^4P_3 \times 4! = \frac{4!}{1!} \times 4! = 4\times3\times2\times 4\times3\times2$

$= 24 \times 24 = 576$ ways

(ii) POUNDING

Vowels = OUI Consonants = PNDNG

n = 8

1 2 3 4 5 6 7 8

4 posn to arrg 3 vowels, N repeating for two times

$\frac{^4P_3 \times 5!}{2!} \Rightarrow \frac{4! \times 5!}{2!} \Rightarrow \frac{4\times3\times2\times1 \times 5\times4\times3\times2}{2!}$

$12 \times 120 = 1440$

Q3. Solve $a_n - 8a_{n-1} + 21a_{n-2} - 18a_{n-3} = 0$

Q4 $a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-3}$ with the initial conditions $a_0 = 9$, $a_1 = 6$, $a_2 = 26$

Q3. $a_n - 8a_{n-1} + 21a_{n-2} - 18a_{n-3} = 0$

It is homogeneous

Step 1: put $a_n = r^n$ $a_{n-1} = r^{n-1}$

$\boxed{a_{n-2} = r^{n-2}}$

$r^n - 8r^{n-1} + 21r^{n-2} - 18r^{n-3} = 0$

Step 2: find an equation in terms of r this is called characteristic equation. ~~and find~~

$r^n\left[1 - \frac{8}{r} + \frac{21}{r^2} - \frac{18}{r^3}\right] = 0$

$r^n\left[\frac{r^3 - 8r^2 + 21r - 18}{r^3}\right] = 0$

$r^3 - 8r^2 + 21r - 18 = 0$

Step 3: find root of the Character equation

$r = 2$

put $r=2$

$(2)^3 - 8(2)^2 + 21(2) - 18 = 0$

$8 - 32 + 42 - 18 = 0$

$50 - 50 = 0$

first root will be $r=2$

2	1	−8	21	−18
	↓	2	−12	18
	1	−6	9	x(0)

So now quadratic eqⁿ

$x^2 - 6x + 9 = 0$

$x^2 - 3x - 3x + 9 = 0$

$x(x-3) - 3(x-3) = 0$

$\boxed{x = 2, 3, 3}$ roots are = 2, 3, 3

Since two roots are same and one root is diff then the general solution will be

$$\boxed{a_n = (b_1 + n b_2) r_1^n + b_3 r_2^n}$$

ans because initial conditions are not given.

Q4 $a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-3}$ with the initial conditions $a_0 = 8$, $a_1 = 6$, $a_2 = 26$

$a_n + a_{n-1} - 4a_{n-2} - 4a_{n-3} = 0$

it is homogeneous

put $r^n = a_n$

$r^n + r^{n-1} - 4r^{n-2} - 4r^{n-3} = 0$

$r^n\left[1 + \frac{1}{r} - \frac{4}{r^2} - \frac{4}{r^3}\right] = 0$

$r^n\left[\frac{r^3 + r^2 - 4r - 4}{r^3}\right] = 0$

$r^3 + r^2 - 4r - 4 = 0$

$r^3 + r^2 - 4r - 4 = 0$

$\boxed{r = -1}$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -4 & -4 \\ & \downarrow & -1 & 0 & +4 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$x^2 - 4 = 0 \qquad x = -2, +2$

roots are $\Rightarrow \boxed{r \Rightarrow 2, -1, -2}$

since all roots are distinct so the general solution will:

$$\boxed{a_n = b_1 r_1^n + b_2 r_2^n + b_3 r_2^n}$$

since the initial conditions are $a_0 = 8,\ a_1 = 6,\ a_2 = 26$

$a_0 = b_1 r_1^0 + b_2 r_2^0 + b_3 r_2^0$

$\boxed{8 = b_1 + b_2 + b_3}$ — (1)

$a_1 = b_1 r_1^1 + b_2 r_2^1 + b_3 r_2^1$

$6 = b_1(2) + b_2(-1) + b_3(-2)$

$6 = 2b_1 - b_2 - 2b_3$ — (2)

$a_2 = b_1 r_1^2 + b_2 r_2^2 + b_3 r_2^2$

$26 = b_1(2)^2 + b_2(-1)^2 + b_3(-2)^2$

$26 = 4b_1 + b_2 + 4b_3$ — (3)

$26 = 4(b_1 + b_3) + b_2$

$\boxed{b_2 = 26 - 4(b_1 + b_3)}$ — (4)

Put eq (4) in eq (1)

$8 = b_1 + 26 - 4(b_1 + b_3) + b_3$

$8 = b_1 + 26 - 4b_1 - 4b_3 + b_3$

$8 = b_1 + 26 - 4b_1 - 3b_3$

$8 - 26 = -3b_1 - 3b_3$

$\cancel{-}18 = \cancel{-}3(b_1 + b_3)$

$\boxed{b_1 + b_3 = 6}$ — (5)

put eq (5) in eq (4)

$b_2 = 26 - 4(6)$

$b_2 = 26 - 24 \qquad \boxed{b_2 = 2}$ — (6)

from eq (2)

~~$6 = 2(b_1 - b_3)$~~ $- b_2$

$6 = 2(b_1 - b_3) - b_2$

$6 = 2(b_1 - b_3) - 2$

$6 = 2b_1 - 2b_3 - 2$

$6 + 2 = 2b_1 - 2b_3$

$8 = 2b_1 - 2b_3$

$\cancel{8} = \cancel{2}(b_1 - b_3)$

$b_1 - b_3 = 4$ —— (7)

from eq (5) & (7)

$b_1 + b_3 = 6$

$b_1 - b_3 = 4$

$2b_1 = 10$

$\boxed{b_1 = 5}$ $\boxed{b_2 = 2}$ put b_1, b_2 in eq^n (1)

$8 = 5 + 2 + b_3$

$b_3 = 8 - 7$ $\boxed{b_3 = 1}$

put b_1, b_2 and b_3 in general solution.

$a_n = 5(2)^n + 2(-1)^n + 1(-2)^n$

$\boxed{a_n = 10^n - 2^n - 2^n}$ answer

Q. In how many ways can the letters of the word "diverse" be arranged so that the vowels never come together.

ans => vowels never come together = (total arrangement) − (arrangement of words where vowels together).

(1) total arrangement = $\frac{7 \text{ word}!}{\text{repeated } w!}$

$= \frac{7!}{2!} \Rightarrow \frac{7\times6\times5\times4\times3\times2\times1}{2\times1} = 2520$

(ii) No of ways to arrange the vowels together:

diverse $n = 7$

vowels = i e e repeated = ee (2)

constants = d v r s (4)

$\frac{5! \times 3!}{2!} \Rightarrow \frac{5\times4\times3\times2\times3\times2\times1}{2\times1} = 360$

ways with vowels never together

$\Rightarrow 2520 - 360 = 2160$ ways

Q. If a set B have n no of elements then what is the total no of subset of B Justify your ans =>

Each ele in set B has two possibilities

It can either be included in a subset or excluded. Therefore, for each ele, there are two choices.

Since there are n elements and each has 2 choices (include & exclude), the total no of possible subsets are.

$$2 \times 2 \times 2 \times \ldots = 2^n$$

MST-2

It is given that white tiger population is 30 at time zero and 32 at time one, also increase from the time $(n-1)$ & time n is twice the increase from the time $n-2$, $n-1$ write recurrence relation for growth rate of the tiger.

$t_0 = 30, \; t_1 = 32$

$n-1 \rightarrow n = 2(n-2) - n(-1)$

$\cancel{-t_{n-1} \; t_n = 2(t_{n-2} - t_{n-1})}$

$-t_{n-1} + t_n = 2(-t_{n-2} + t_{n-1})$

$t_n = 3t_{n-1} - 2t_{n-2}$

$t_n - 3t_{n-1} + 2t_{n-2} = 0$

$r^n - 3r^{n-1} + 2r^{n-2} = 0$

$r^n \left[1 - \frac{3}{r} + \frac{2}{r^2} \right] = 0$

$r^2 - 3r + 2 = 0$

$\cancel{x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times (-2)}}{2 \times 1}}$

$\cancel{x = \frac{3 \pm \sqrt{9+8}}{2}} \Rightarrow \frac{3 \pm \sqrt{17}}{2}$

$x =$

$r^2 - 3r + 2 = 0$

$r^2 - 2r - r + 2 = 0$

$r(r-2) - 1(r-2) = 0$

$\boxed{r = 2, 1}$

$t_n = b_1 r_1^n + b_2 r_2^n$

$\boxed{t_n = b_1 (1)^n + b_2 (2)^n}$

$t_0 = 30$
$t_1 = 32$

$30 = b_1 (1)^0 + b_2 (2)^0$

$30 = b_1 + b_2 \quad — (1)$

$32 = b_1(1)^1 + b_2(2)^1$

$32 = b_1 + 2b_2$ —— (2)

$\boxed{b_1 = 32 - 2b_2}$ put in

(1) - (2)

$30 = b_1 + b_2$

$32 = b_1 + 2b_2$

$\therefore -2 = -b_2$

$\boxed{b_2 = 2}$ put in (1)

$\boxed{b_1 = 28}$

$$\boxed{t_n = 28(1)^n + 2(2)^n}$$

* Method of characteristic roots for non Homogeneous linear Reccurence relation with constant Coefficient.

$f(n) \neq 0$ → homogeneous solution, particular solution

$$\boxed{a_n = a_n^{(h)} + a_n^{(p)}}$$

General sol^n is a combination of homogeneous sol^n and of particular sol^n.

case

(i) a_n = Constant i.e. $f_n = \alpha$ (constant)

* Steps to find $a_n^{(p)}$

(i) let $a_n = A$

(ii) Put $a_n = a_{n-1} = a_{n-2} = A = A$ in the given reccurence relation.

(iii) find the value of A.

(iv) $\boxed{a_n^{(p)} = A}$

Q. Solve

$a_{n+2} - 5a_{n+1} + 6a_n = 2$ with initial condition $a_0 = 1$, $\boxed{a_1 = 1}$ To find (a_n^h)

$a_n^{(h)} \Rightarrow \boxed{b_1(r_1)^n + b_2(r_2)^n}$

$a_{n+2} - 5a_{n+1} + 6a_n = 0$

$a_n = r^n$

$r^2 - 5r + 6 = 0$

$r^2 - 3r - 2r + 6 = 0$

$r(r-3) - 2(r-3) = 0$

$r = 2, 3$

$$\Rightarrow \boxed{a_n = b_1(3)^n + b_2(2)^n}$$

$a_0 \Rightarrow b_1 + b_2 = 1$ — (1)
$a_1 \Rightarrow 3b_1 + 2b_2 = -1$ — (2)

Multiply (1) by 2 & Subtract
(2) − (1)

$\Rightarrow \quad 3b_1 + \cancel{2b_2} = -1$
$\quad\; 2b_1 + \cancel{2b_2} = 2$

$\boxed{b_1 = -3}$ ✓

From (1) $\boxed{b_2 = 4}$ ✓

Now, particular Solution

$a_n^p = a_{n+2} - 5a_{n+1} + 6a_n = 2$. Let $a_n = A$

$A - 5A + 6A = 2$

$\boxed{A = 1}$

a_n = homogeneous + particular Sol^n

$$\boxed{a_n = (-3)(3)^n + 4(2)^n + 1}$$

Q. $a_n - 6a_{n-1} + 8a_{n-2} = 3$ $\quad 1 - 6r^{-1} + 8r^{-2} = 0$

$r^2 - 6r + 8 = 0$

$r^2 - 2r - 4r + 8 = 0$

$r(r-2) - 4(r-2) = 0 \quad r = 2, 4$

a_n

$a_n^{(h)} \Rightarrow a_n = b_1(4)^n + b_2(2)^n \rightarrow$ Homogeneous Sol^n

$(p) \rightarrow a_n - 6a_{n-1} + 8a_{n-2} = 3 \Rightarrow A - 6A + 8A = 3$

$a_n \Rightarrow \boxed{A = 1}$ $\rightarrow 9A - 6A = 3$, $3A = 3$

$\boxed{a_n = b_1(2)^n + b_2(4)^n + 1}$

$f(n) = C_0 a_n + C_1 a_{n-1} + C_2 a_{n-2} + \dots C_k a_{n-k}$

$f(n) = p(n)$

Case 2 ⇒

(1) $P(n)$ is a polynomial of degree s.

$a_n = a_n^h + a_n^p$

* Steps to find $(a_n)^p$

(1) Let a_n is equal to

$a_n = A_0 + A_1 n + A_2 n^2 + \dots A_s n^s$

(2) Put the value of $a_n, a_{n-1}, a_{n-2} \dots$ in the given equation

(3) Compare the coefficients of like powers on n.

(4) Find the value of $A_0, A_1, A_2 \dots A_s$

(5) $\boxed{a_n^{P} = A_0 + A_1 n + A_2 n^2 + \dots A_s n^s}$

$y_{n+2} - y_{n+1} - 2y_n = n^2$

general solution $\rightarrow$ $\boxed{a_n = a_n^{h} + a_n^{P}}$

$a_n^{h} \Rightarrow y_n = r^n$

$r^2 - r - 2 = 0$ (Homogeneous sol^n)

$\boxed{r = -1, 2}$

= general sol^n $\Rightarrow a_n = b_1 (r_1)^n + b_2 (r_2)^n$

$a_n = b_1(-1)^n + b_2(2)^n$

(P) $y_{n+2} - y_{n+1} - 2y_n = n^2$ degree = 2

$\Rightarrow a_n^{P} \Rightarrow y_n = A_0 + A_1 n + A_2 n^2$

$y_n = \{A_0 + A_1(n+2) + A_2(n+2)^2\} - \{A_0 + A_1(n+1) + A_2(n+1)^2\} - 2\{A_0 + A_1 n + A_2 n^2\} = n^2$

$\{A_0 + A_1(n+2) + A_2(n^2 + 4 + 4n)\} - \{A_0 + A_1(n+1) + A_2(n^2 + 1 + 2n)\} - 2\{A_0 + A_1 n + A_2 n^2\} = n^2$

$\{A_0 + A_1 n + 2A_1 + n^2 A_2 + 4A_2 + 4nA_2\} - \{A_0 + A_1 n + A_1 + A_2 n^2 + A_2 + 2nA_2\} - 2A_0 - 2A_1 n - 2A_2 n^2\} = n^2$

$\cancel{A_0} + \cancel{A_1 n} + 2A_1 + \cancel{n^2 A_2} + 4A_2 + 4nA_2 - \cancel{A_0} - \cancel{A_1 n} - A_1 - \cancel{A_2 n^2} - A_2 - 2nA_2 - 2A_0 - 2A_1 n - 2A_2 n^2$

$A_1 + 3A_2 + 2nA_2 - 2A_0 - 2A_1 n - 2A_2 n^2 = n^2$

$A_1 + 3A_2 + 2nA_2 - 2A_0 - 2A_1 n - 2A_2 n^2 = n^2$

$(3A_2 + A_1 - 2A_0) + (2A_2 - 2A_1)n - 2A_2 n^2 = n^2$

compare coefficients.

$3A_2 + A_1 - 2A_0 = 0$ —— (1)

$2A_2 - 2A_1 = 0$ —— (2)

$-2A_2 = 1$ —— (3)

$\boxed{A_2 = \frac{-1}{2}}$

From (2) $\boxed{A_1 = \frac{-1}{2}}$

From (1) $\boxed{A_0 = -1}$

$\boxed{a_n = b_1(2)^n + b_2(-1)^n - 1 - \frac{1}{2}n - \frac{1}{2}n^2}$

(1) $a_n = 2a_{n-1} - a_{n-2} + 2^n + 2$

$a_1 = 7,\ a_2 = 19$

(2) $x_n = 6x_{n-1} - 12x_{n-2} + 8x_{n-3}$

$x_0 = -1,\ x_1 = 0,\ x_2 = 1$

$x_n = 6x_{n-1} - 12x_{n-2} + 8x_{n-3}$

$x_n - 6x_{n-1} + 12x_{n-2} - 8x_{n-3} = 0$

$r^n - 6r^{n-1} + 12r^{n-2} - 8r^{n-3} = 0$

$r^3 - 6r^2 + 12r - 8 = 0 \quad \boxed{r=2}$

$r-2 \,|\, r^3 - 6r^2 + 12r - 8 \,|\, (r^2 - 4r + 4)$

$r^3 - 2r^2$

$-4r^2 + 8r$

$-4r^2$

$4r - 8$

$(r-2)(r^2 - 4r + 4) = 0$

$(r-2)(r-2)(r-2) = 0$

$r = 2, 2, 2$

$a_n = (b_1 + b_2 n + b_3 n^2) r_1^n$

$a_0 = b_1 = -1$ — (1)

$a_1 = (b_1 + b_2 + b_3)\, 2 = 0$

$b_1 + b_2 + b_3 = 0$

$b_2 + b_3 = 1$ — (2)

$a_2 = (b_1 + 2b_2 + 4b_3)(4) = 1$

$b_1 + 2b_2 + 4b_3 = \frac{1}{4}$

$2(b_2 + b_3 + b_3) = \frac{5}{4}$

$1 + b_3 = \frac{5}{8}$

$\boxed{b_3 = \frac{-3}{8}}$

$\boxed{b_2 = \frac{11}{8}}$

$\boxed{a_n = \left(-1 + \frac{11}{8}n - \frac{3}{8}n^2\right)(2)^n}$

* Generating functions

$a_0, a_1, a_2, \ldots a_n$

$G(x) = a_0x^0 + a_1x^1 + a_2x^2 + \ldots a_nx^n + \ldots$

A generating function is a way to mathematically write a sequence of a mathematical expression, it allows the sequence to be mathematically manipulated

$$\boxed{G(x) = \sum_{n=0}^{\infty} a_n x^n}$$

if I have → 1, 1, 1, 1, 1, 1, 1, 1, 1 - - -

$1 \cdot x^0 + 1 \cdot x^1 + 1 \cdot x^2 + 1 \cdot x^3 + \ldots$

$\Rightarrow 1 + x + x^2 + x^3 + \ldots$

$\boxed{G.P = \frac{a}{1-r}} = \frac{1}{1-\left(\frac{x}{1}\right)} \left(\frac{1}{1-x}\right)$

Q. 0, 1, 1, 1, 1

$0 \cdot x^0 + 1 \cdot x^1 + 1 \cdot x^2 + 1 \cdot x^3 + \ldots$

$G(x) = x + x^2 + x^3 + \ldots$

$G(x) = x(\underbrace{1 + x + x^2 + \ldots}_{G.P})$ $\quad G.P = \frac{a}{1-r}$

$$\boxed{G(x) = x \cdot \frac{1}{1-x}}$$

Q. 0, 1, 2, 3, 4

$G(x) = 0 \cdot x^0 + 1 \cdot x^1 + 2 \cdot x^2 + 3 \cdot x^3 + 4 \cdot x^4$

$\boxed{G(x) = x + 2x^2 + 3x^3 + 4x^4}$ —— (1)

$x(G(x)) = x^2 + 2x^3 + 3x^4 + 4x^5$ —— (2)

sub eqn (1) − (2)

$G(x) - x(G(x)) = x + (2x^2 - x^2) + (3x^3 - 2x^3) + (4x^4 - 3x^4) + 0 \ldots$

$= x + x^2 + x^3 + x^4 + \ldots$

$= x(1 + x + x^2 + x^3 + \ldots)$

$$G(x) - xG(x) = x\left(\frac{1}{1-\cancel{x}}\right) \cdot \frac{x}{1-x}$$

$$G(x)\left[(1-x)\right] = \frac{x}{(1-x)}$$

$$\boxed{G(x) = \frac{x}{(1-x)^2}}$$

Q $1, 0, -1, 0, 1, 0, -1 \ldots$

$$G(x) = 1 \cdot x^0 + 0 \cdot x^1 + (-1)(x^2) + 0(x^3) + 1 \cdot x^4 + 0 \cdot x^5 + (-1)(x^6)$$

$$\cancel{G(x) = x + 0 + x + 0 + x + 0 + x}$$

$$\cancel{G(x) = x + x + x + x}$$

$$G(x) = 1 - x^2 + 0 + x^4 + 0 - x^6$$

$$\boxed{G(x) = 1 - x^2 + x^4 - x^6 + \ldots}$$

$$G(x) = 1 + (-x^2) + (-x^2)^2 + (-x^2)^3$$

$$G_P = \frac{1}{1-(-x^2)} = \frac{1}{1+x^2} \quad \Big| \; r = (-x^2), \; r = -x^2$$

$$\boxed{G(x) = \frac{1}{1+x^2}}$$

a_k	$G(x)$
1	$\frac{1}{1-x}$
$(-1)^k$	$\frac{1}{1+x}$
c	$\frac{c}{1-x}$
$k+1$	$\frac{1}{(1-x)^2}$
k	$\frac{x}{(1-x)^2}$
a^k	$\frac{1}{1-ax}$
$(-a)^k$	$\frac{1}{1+ax}$
k^2	$\frac{x(1+x)}{(1-x)^2}$
ka^k	$\frac{ax}{(1-ax)^2}$

Q1 $a_k = 6^k$, Q2. $a_k = k+1$

Q3. 1, 1, 0, 1, 1, 1, ...

Q4 7, −7, 7, −7, 7, ...

Q5 $a_k = 16^k = 6\left(\frac{1}{1-x}\right)$

Q6 $a_k = 5 + 7k$

* Solution of Recurrence Relation using generating function.

Step 1: In given equation multiply by x^k where k is your variable

Step 2: Take summation from $k = 1$ to ∞ if one initial condition is given

if 2 initial conditions are given → then take summation from 2 to ∞

Step 3: Write each summation in terms of $G(x)$ or closed form.

Step 4: put the value of each summation and find value of x

Step 5: find partial fractions of $G(x)$

6: write a_k.

$G(x) = a_0 + a_1 x + (a_2 x^2 + a_3 x^3 + \ldots) \ldots$

Q. Solve $a_r - a_{r-1} - 2a_{r-2} = 0$

$a_0 = 0,\ a_1 = 1$

Step 1: Multiply by x^r

$\sum a_r \cdot x^r - a_{r-1} x^r - 2a_{r-2} x^r = 0 \cdot x^r$

Step 2: $\underbrace{\sum_{r=2}^{\infty} a_r x^r} - \underbrace{\sum_{r=2}^{\infty} a_{r-1} x^r}_{2} - \underbrace{\sum_{r=2}^{\infty} 2a_{r-2} x^r}_{3} = 0$

Step 3: $\sum_{r=2}^{\infty} a_r x^r = a_2 x^2 + a_3 x^3 + a_4 x^4 + \ldots$

$\boxed{\sum_{r=2}^{\infty} a_r x^r = G(x) - a_0 - a_1 x}$

$\boxed{G(x) - a_0 - a_1 x}$

$\sum_{r=2}^{\infty} a_{r-1} x^r = a_1 x^2 + a_2 x^3 + a_3 x^4 + \ldots$

$= x(a_1 x^1 + a_2 x^2 + a_3 x^3 + a_0 - a_0)$

$\Rightarrow x(G(x) - a_0)$

$\Rightarrow$

$$\sum_{r=2}^{\infty} 2a_{r-2} x^r$$

$$\sum_{r=2}^{\infty} 2a_{r-2} \cdot x^r = 2a_0 \cdot x^2 + 2a_1 x^3 + 2a_2 x^4 + \ldots$$

$$= 2x^2 (a_0 \cdot x^0 + a_1 x^1 + a_2 x^2 + \ldots$$

$$= 2x^2 (G(x))$$

$$= (G(x) - a_0 - a_1 x) - x(G(x) - a_0) - 2x^2 (G(x))$$

$$\Rightarrow G(x) - a_0 - a_1 x - x G(x) + x a_0 - 2x^2 G(x) = 0$$

$$G(x) - x G(x) - 2x^2 G(x) = a_0 + a_1 x - x a_0$$

$$G(x) - x(G(x)) - 2x^2 (G(x)) = 0 + x - 0$$

$$G(x)[1 - x - 2x^2] = x$$

$$\boxed{G(x) = \frac{x}{1 - x - 2x^2}}$$

Now partial fraction

$$\Rightarrow -(2x^2 + x - 1)$$

$$\Rightarrow -(2x^2 + 2x - x - 1)$$

$$\Rightarrow -(2x(x+1) - 1(x+1))$$

$$-[(2x-1)(x+1)]$$

$$\Rightarrow G(x) = \frac{x}{(1-2x)(-1-x)} \Rightarrow 1 - 2x = 0,\ 1 = 2x,\ \boxed{x = \frac{1}{2}}$$

$$\Rightarrow \frac{\frac{1}{2}}{-1-(\frac{1}{2})} = \frac{\frac{1}{2}}{\frac{-3}{2}} = \frac{1}{2} \times \frac{2}{-3} = \frac{-1}{3}$$

$$\Rightarrow -1 - x = 0 \Rightarrow \boxed{-1 = x}$$

$$\Rightarrow \frac{-1}{1-(2(-1))} = \frac{-1}{1+2} = \frac{-1}{3} \Rightarrow G(x) = \frac{\frac{-1}{3}}{(1-2x)} + \frac{\left(\frac{-1}{3}\right)}{(-1-x)}$$

$$G(x) = \frac{\frac{-1}{3}}{(1-2x)} + \frac{\frac{1}{3}}{(x+1)} \quad \boxed{G(x) = -\frac{1}{3} \cdot 2^r + \frac{1}{3}(-1)^r}$$

Q. Find the Generating function of the numeric function

$$a_r = 2^r + 3^r \quad \text{for } r \geq 0$$

Q2 Find the generating fun of the sequence

$$a_r = (r+2) \cdot (r+1) 3^r$$

Q3. find the generating function and the sequence of recurrence relation with

$a_n + 2a_{n-1} = 0$ $a_0 = 5$

tute

Q4. $t_n = 2t_{n-1} + 8t_{n-2}$

$n=1, \ t_n = 10$

$n=0, \ t_n = 4$

Q5. solve the recurrence relation using Generating function.

$S_n - 6S_{n-1} + 8S_{n-2} = 0$

$S_0 = 10, \ S_1 = 25$

Q6. $a_{r+2} - 5a_{r+1} + 6a_r = 2$

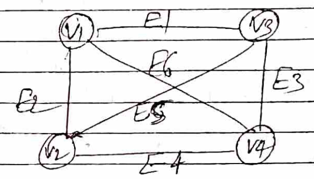
$a_0 = 1, \ a_1 = 2$

Graph Theory

Graph: A Graph G is a mathematical structure consisting of two set V and E where V is the set of vertices and E is a set of edges in a graph.

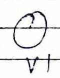
$V = \{V1, V2, V3 \ldots\}$

$E = \{E1, E2, E3 \ldots\}$

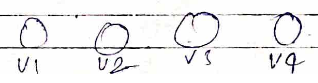


Basic terminologies:

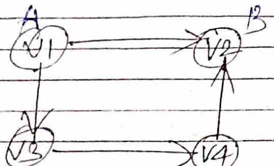
(i) Trivial Graph: A Graph consisting of only one vertex and No edges.



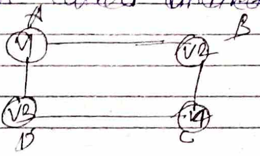
(2) Null Graph: A Graph consisting of n number of vertices and no edges.



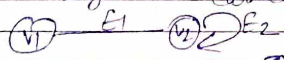
③ Directed Graph: A Graph consist the direction of edges then such a graph called directed graph.



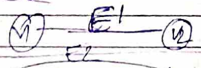
Undirected graph: A Graph consist does not the direction of edges then such a graph called Undirected graph.



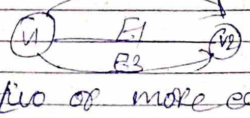
* Self loop: If an edges having same end vertices of it's vertex then that edges are called Self loop.



* Proper Edge:

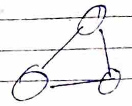


* Multi Edge:

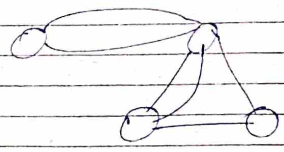


A collection of two or more edges having identical end-points then that is called Multi-Edge.

* Simple Graph:

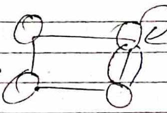


* Multi-Graph: Edge



* Pseudo graph:

self loop + multiedge



* Incidence and adjancency

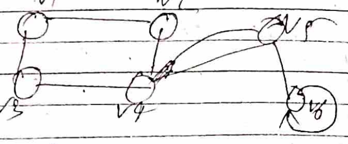
Let Ek be an edge joining two vertices vi & vj then Ek is said to be incidence of vi & vj

Adjancency:

* Two vertices are said to be adjancent If there exist an edge ~~joing~~ joining these vertices

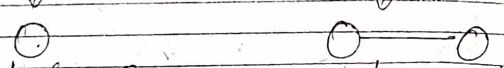
* Degree of vertices: The degree of a vertex V in a graph G, written as d(V) is equal to the no of edges which are incident on V, with self loop counted twice.

Ex ⇒



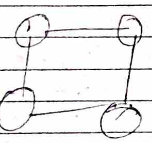
degree of V3 = 2
Vd = 4

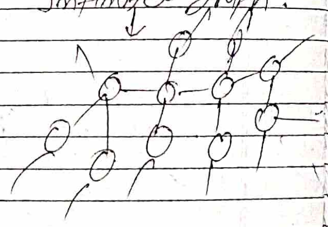
* Isolated vertex and pendant:



degree = 0 degree = 1

* finite & Infinite graph:





* Classification of graph (i) based on loops and multi-edges
- Psuedo Graph
- Multi "
- Simple "

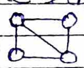
* (ii) Based on the orientation of Edges
(i) undirected graph
(ii) directed graph
(iii) Digraph "

* (3) Based on the weight of the edges:
(i) weighted graph
(ii) unweighted graph

* Eulerean chains & cycles
Eulerean graph = Eulear path + Eulear circuit

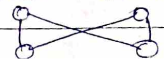
(i) Eulears path: It means that you have to cover all the edges without any repeatation.

Ex ⇒



Eulears circuit or cycle: It means that, you have to start from a point cover all the edges without any repeatation and then reach the same initial point.

Ex:



Hamiltonian Graph = H path + H circuit

* walk Trail path

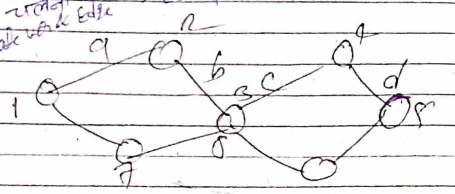
walk → repeat vertex Edge

↳ edge repeat (x)

→ vertex repat (x)

walk ⇒ 1a2b3c4d

Trail ⇒

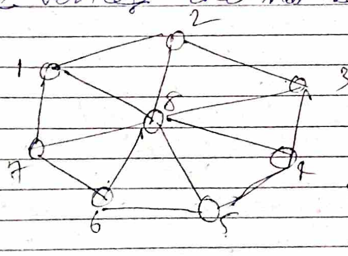


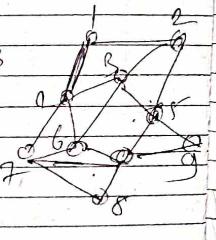
walk = 1a2b3c

walk: walk is a finite alternating Sequence $v_1 E_1, v_2 E_2, v_3 E_3 \ldots E_{n}$ of vertices and edges begining and ending with same and different vertices

length: the no of edges of walk

* A walk is called a path if all the vertices are not repeated



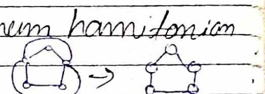


* Hamiltonian Graph:

→ Hamiltonian path: A path which containing every vertex of a graph G exactly once is called Hamiltonian path (No vertices should be repeated).

* Hamiltonian circuit: A circuit that passes through each vertex in a graph G exactly once except the starting and the ending vertex is called hamiltonian circuit.

Q1 Determine a minimum hamiltonian circuit for the graph



Q2 Draw a graph with 6 vertices containing a hamiltonian circuit ~~but not~~ but not Eulerian circuit

Q3 Draw hamiltonian circuit and hamiltonian path

Q4 check whether the graph is hamiltonian graph or not

Q5 give an example of a graph which has

(i) Euler circuit but not hamiltonian circuit

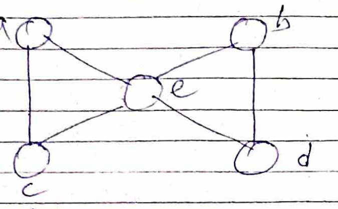
(ii) Neither hamiltonian G and Eulerian circuit

(3) Both ham graph & eular graph

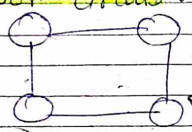
Hamiltonian Graph = H path + H circuit

Q3 → * Hamiltonian path : Example

Q4



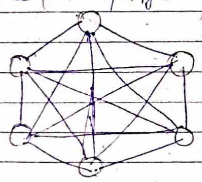
Hamiltonian circuit : Example

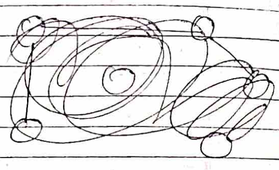


Hamiltonian Graph & circuit

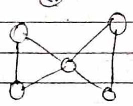
py Q Dift b/w Eular and Hamilton Graph

Q2. before page Answer →

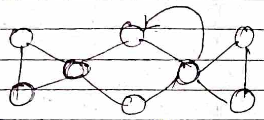




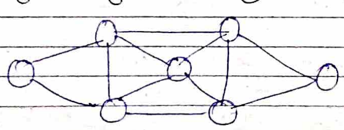
Q5 (i)



(ii)

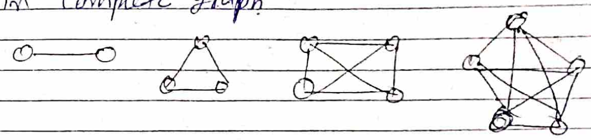


(iii)



py Q

* Complete graph: A simple graph of 'n' vertices having exactly one edge b/w each pairs of vertices is called a complete graph. A complete graph of 'n' vertices is denoted as 'Kn' total no of edges $\boxed{\frac{n(n-1)}{2}}$ with 'n' vertices in complete graph.

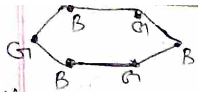


* Graph Coloring: The assign of colors to the vertices of G, one color to each vertex so that adjacent vertices are assigned different color is called the proper coloring of G of G or simply vertex coloring.

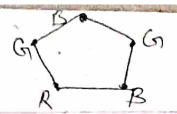
* Cromatic number: Cromatic number of a Graph G is the minimum number of colors to color the vertices of the graph G and is denoted by $\chi(G)$

1. if $\chi(G) = K$ then the graph is 'K' Cromatic
2. The Cromatic no of null graph is 1
3. The Cromatic no of complete graph

Ex)



Chromatic no = 2

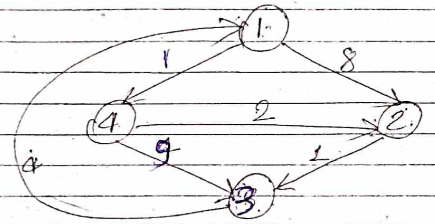


Cr no = 3

G with n vertices is 'n'

- If a graph is circuit with 'n' vertices then
- It is 2-Chromatic, if n is even
- It is 3-chromatic, if n is odd.

* flayed warshall algorithm



Step 1:

$$D_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & \infty & 1 \\ 2 & \infty & 0 & 1 & \infty \\ 3 & 4 & \infty & 0 & \infty \\ 4 & \infty & 2 & 9 & 0 \end{array}$$

$$D_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & \infty & 1 \\ 2 & \infty & 0 & 1 & \infty \\ 3 & 4 & 12 & \infty 0 & 5 \\ 4 & \infty & 2 & 9 & 0 \end{array}$$

0, 1
direct

direct or root/source node or through
(0, 1)
Node, Node

direct, Nod 1 / Node 2, sor
combine
$\boxed{0, 1, 2, (1, 2)}$

$$D_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & 9 & 1 \\ 2 & \infty & 0 & 1 & \infty \\ 3 & 4 & 12 & 0 & 5 \\ 4 & \infty & 2 & 3 & 0 \end{array}$$

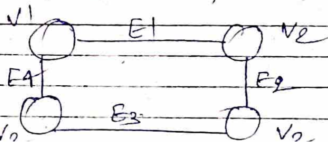
$$D_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & 9 & 1 \\ 2 & 5 & 0 & 1 & 6 \\ 3 & 4 & 12 & 0 & 5 \\ 4 & 7 & 2 & 3 & 0 \end{array}$$

$$D_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 4 & 1 \\ 2 & 5 & 0 & 1 & 6 \\ 3 & 4 & 7 & 0 & 5 \\ 4 & 7 & 2 & 2 & 0 \end{array}$$

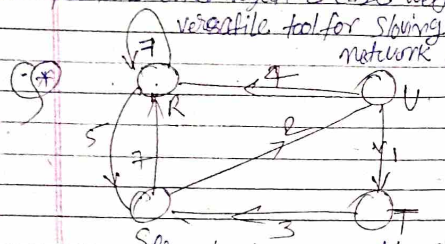
* Simple Graph: A graph that does not contain following of them

(i) No loop
(ii) undirected edge
(iii) No multi-edge
(iv) finite no of vertices.

Ex ⇒



* Floyd warshall Algorithm: The floyd-warshall Algorithm, named after its creators Robert floyd and stephen warshall is a fundamental Algorithm in computer science and graph theory. It is used to find the shortest paths between all pairs of nodes in a weighted graph. This algorithms is highly efficient and can handle graphs with both positive and negative edge weights, making it a versatile tool for solving a wide range of network and connectivity problems.



Step1: initially the distance matrix using the input graph such that distance = weight of edge from i to j. also distance = ∞ if there is no edge from i to j.

$$D_0 = \begin{array}{c|cccc} & R & S & T & U \\ \hline R & 7 & 5 & \infty & \infty \\ S & 7 & 0 & \infty & 2 \\ T & \infty & 3 & 0 & \infty \\ U & 4 & \infty & 1 & 0 \end{array}$$

$$D_1 = \begin{array}{c|cccc} & R & S & T & U \\ \hline R & 7 & 5 & \infty & \infty \\ S & 7 & 0 & \infty & 2 \\ T & \infty & 3 & 0 & \infty \\ U & 4 & 9 & 1 & 0 \end{array}$$

step 2: Treate Node 1 as an intermidiate node and calculate the distance

Step 3 => Treate Node 1 and 2 or combination of 1 and 2 as an intermidiate node and calculate the distance

$$D_2 = \begin{bmatrix} 7 & 5 & \infty & 7 \\ 7 & 0 & \infty & 2 \\ 10 & 3 & 0 & 5 \\ 9 & 9 & 1 & 0 \end{bmatrix}$$

Step 4: Treate Node 1, 2, 3 and combination of the nodes and calculate distance.

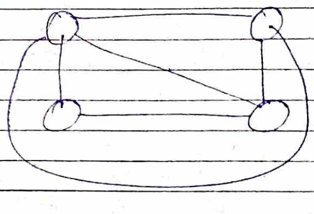
$$D_3 = \begin{bmatrix} 7 & 5 & \infty & 7 \\ 7 & 0 & \infty & 2 \\ 10 & 3 & 0 & 5 \\ 9 & 4 & 1 & 0 \end{bmatrix}$$

Step 5: Treate Node 1, 2, 3, 4 and combination of it and calculate distance.

$$D_4 = \begin{bmatrix} 7 & 5 & 8 & 7 \\ 8 & 0 & 3 & 2 \\ 9 & 3 & 0 & 5 \\ 9 & 4 & 1 & 0 \end{bmatrix}$$

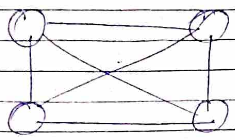
* Planar's Graph: A Graph is called a planar graph if it can be drawn without any crossing edge.

Ex =>

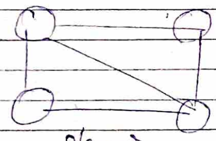


* Non planar Graph: A graph is called non planar if it can be drawn with any crossing edges.

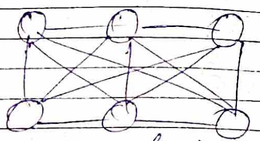
Ex →



Note → Planar graph is possible for complete graph only if vertices are less than 5.



planar

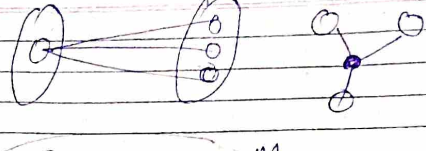


Non planar

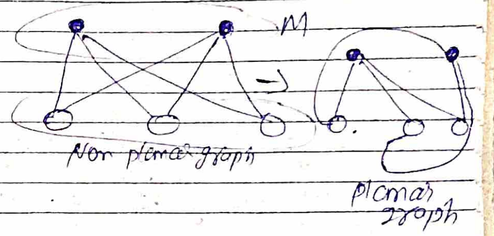
(*) W Bipartite Graph: A graph G is said to be bipartite, if it's vertices V can be partioned into 2 subsets M & N such that each edge of G connects a vertices of M to a vertex of N. A complete bipartite graph means that each vertex of M is connected to each vertex of N. This graph is donated by $K_{m,n}$, where m is the no of vertices of M and n is the no of vertices in N. $(m \le n)$

Example M N

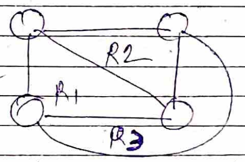
$K_{1,3}$ →



$K_{2,3}$ →



* Regions in a planar Graph:



Region is the area that is bounded by edges.
R_1, R_2, R_3 → bounded
R_4 → unbounded

The planar representation of a graph splits the plain in regions. These region are bounded by edges but for one region which is unbounded.

HW → Handshaking Theorem
Euler's theorem for planar Graph

$$v - e + f = 2$$

v → vertices, e → edge, f → faces

* Properties of planar Graph:

(i) If a connected planar graph G has 'e' edges & 'r' regions then

$$\boxed{r \leq \frac{2}{3} e}$$

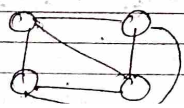
(ii) If a connected planar graph G has 'e' edges 'v' vertices & 'r' regions then $\boxed{v - e + r = 2}$

(iii) If a connected planar Graph G has 'e' edges & 'v' vertices then $\boxed{3v - e \geq 6}$

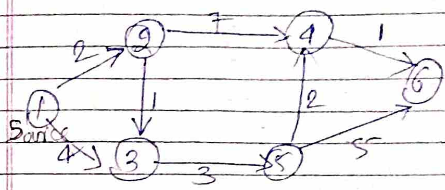
(iv) A complete graph K_n is planar, if & only if $n < 5$

(v) A complete bipartite graph $K_{m,n}$ is planar if & only if $\boxed{m \leq 3}$ or $\boxed{n > 3}$

Q. Prove that a complete graph K_4 is planar or not.



* Dijkstra's Algorithm

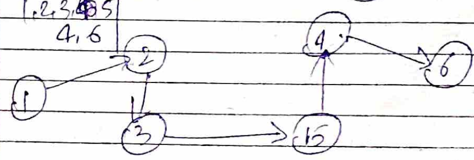


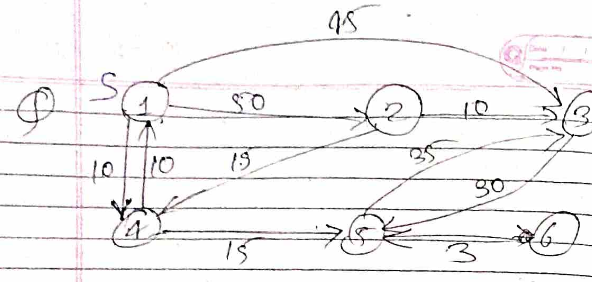
Dijkstra's Algo : If a weighted graph is given then we have to find the shortest path b/w different vertices from any source vertex

$\boxed{d(U) + c(U,V) < d(V)}$ [define]

$\boxed{d(V) = d(U) + c(U,V)}$

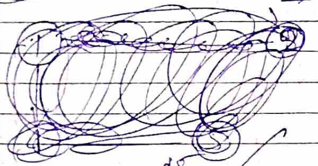
Selected vertex	2	3	4	5	6
1	∞	∞	∞	∞	∞
1	2	4	∞	∞	∞
1, 2	2	3	9	∞	∞
1, 2, 3	2	3	9	6	∞
1, 2, 3, 5	2	3	8	6	11
1, 2, 3, 4, 5					
4, 6					

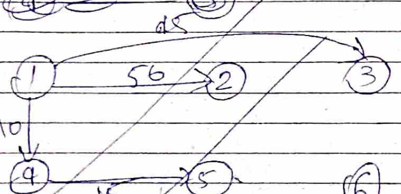


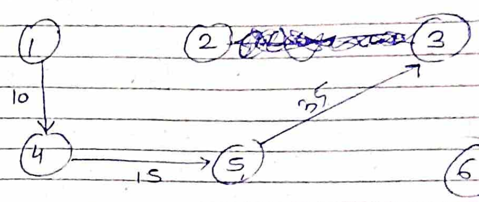


	1	2	3	4	5	6
1	∞	∞	∞	∞	∞	∞
1, 4		50	45	10	∞	∞
1, 4, 5		50	45	10	25	∞
~~1, 4, 5,~~ 6		50	45	10	25	∞
1, 4, 5, 6, 3		50	45	10	28	∞
1, 4, 5, 3, 2		50	45	10	28	∞

, 6



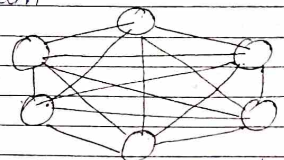




	2	3	4	5	6
1	∞	∞	∞	∞	∞
1, 4	50	45	10	∞	∞
1, 4, 5	50	45	10	25	∞

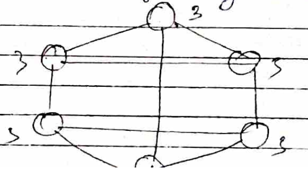
PYQ Connected Graph: A connected graph is a graph where every pair of vertices is connected by a path.

Example:



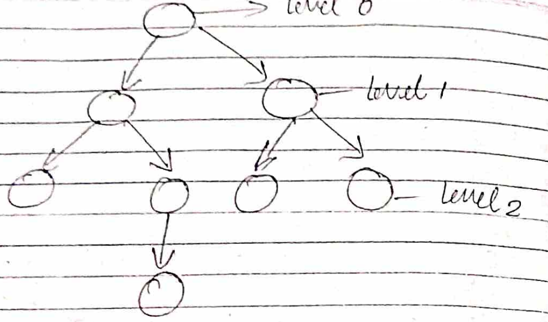
PYQ Regular Graph: A regular graph is a graph where each vertex has the equal no of neighbour means every vertex has same degree.

Ex =>



every vertex has 3 equal degrees.

Tree



Tree : A tree is a connected acyclic undirected graph, there is a unique path b/w everywhere of vertices in Graph; A ~~t~~ tree is a discrete structure that represents hierarchical relationship b/w individual elements or nodes

$\boxed{n \text{ vertex},\ n-1 \text{ edges}}$

* Properties of ~~Spanning~~ Tree

(i) There is only one path b/w each pair of vertices in a tree

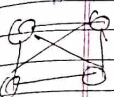
(ii) A ~~tree~~ T with n vertices has n-1 edges.

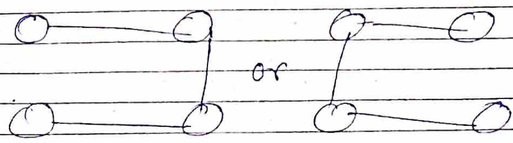
(3) A Graph is a tree if and only if it is minimually connected.

* Spanning Tree : A connected sub-graph S of a graph G(V,E) is said to be spanning if and only if S should contain all the vertices of the graph G.

(ii) S should contain (|V|-1) edges.

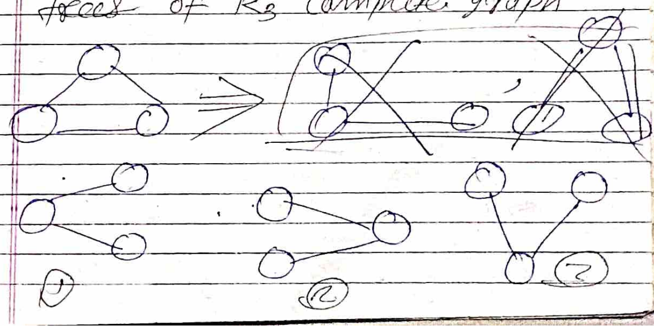
Eg =>





Q Draw all the possible spanning trees of K_3 complete graph

3^{3-2}
3
$= 3$

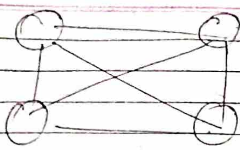


Ex) Construct all the possible spanning trees of K4 complete graph.

(ii)

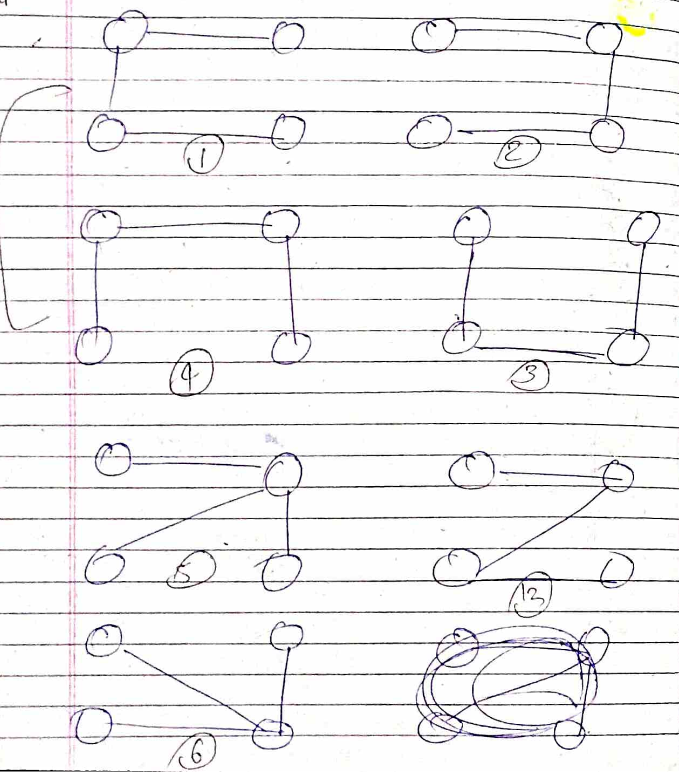
4^{4-2}

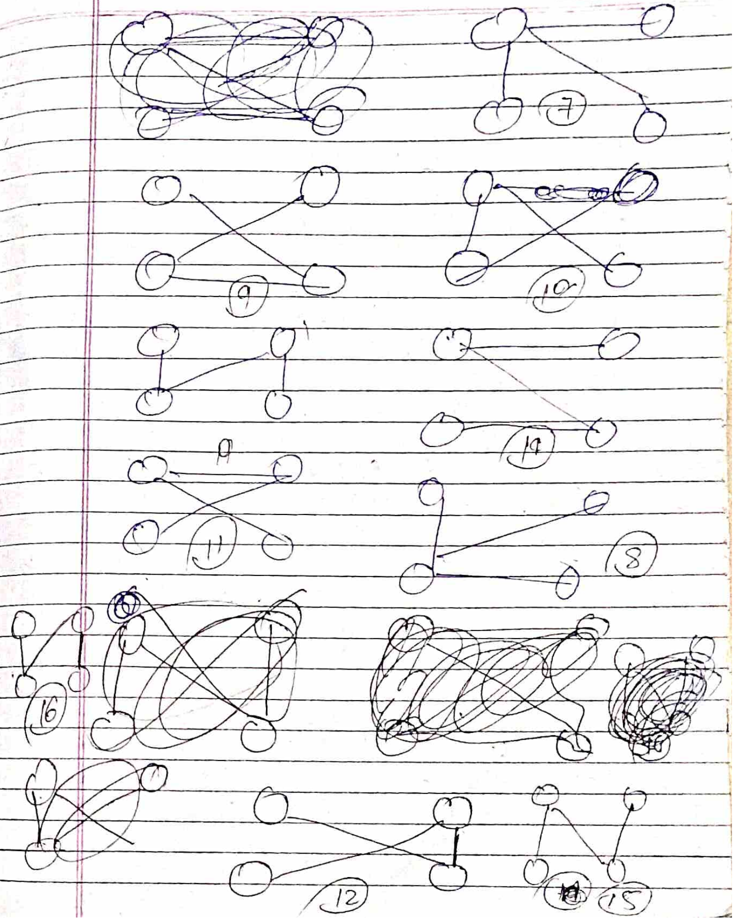
$= 4^2 = 16$



n vertices

ST → n^{n-2}





(*) Minimum Spanning Tree: A Spanning Tree with assigned weight less than or equal to the weight of every possible spanning tree of the a weighted connected and undirected graph is called a minimum spanning tree. (MST).

The weight of a minimum spanning tree is the sum of all the weights assigned to each edge of a spanning tree.

* Kruskal's Algorithm: It is a griddy algorithm that finds a minimum spanning tree for a connected weighted graph. It finds a tree of that graph which includes every vertex and the total weight of all the edges in the tree is less than or equal to every possible spanning tree.

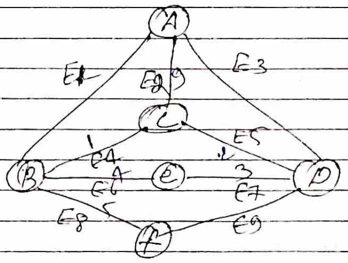
Step 1: Arrange all the edges of the given graph G(V, E) in ascending order as per their edge weight

Step 2: Choose the smallest weighted edge from the graph and checks if it forms a cycle with a spanning tree form so far.

Step 3: If there is no cycle include this edge to the spanning tree else discard it.

Step 4: Repeat step (2) and (3) until (V−1) no of edges are left in the spanning tree.
V → vertices.

(1)

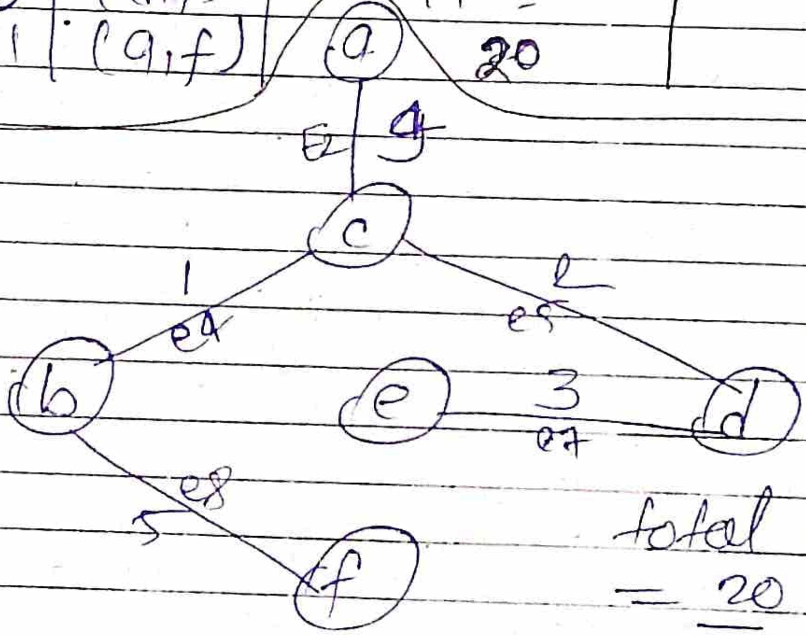


step1 :

e1	(a,b)	20
e2	(a,c)	9
e3	(a,d)	13
e4	(c,b)	1
e5	(c,d)	2
e6	(b,e)	4
e7	(e,d)	3
e8	(b,f)	5
e9	(d,f)	11

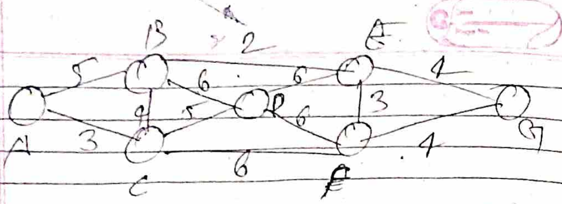
step2 : Ascending order

e4	(b,c)	1
e5	(c,d)	2
e7	(e,d)	3
e6	(b,e)	4
e8	(b,f)	5
e2	(a,c)	9
e3	(a,d)	13
e9	(d,f)	14
e1	(a,f)	20

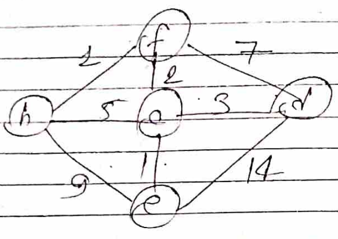


total edge = 20

Q1



Q2



* Dijkstra Algorithm:

A Prim's Algorithm: Prim's algorithm is a greedy algorithm that finds a minimum spanning tree for a connected weighted graph. It finds a tree of that graph which includes every vertex and the total weight of all the edges in the tree is less than or equal to every possible tree. It is faster on dense graph.

Step1: initialize the minimum spanning tree with a single vertex randomly choosen from the graph

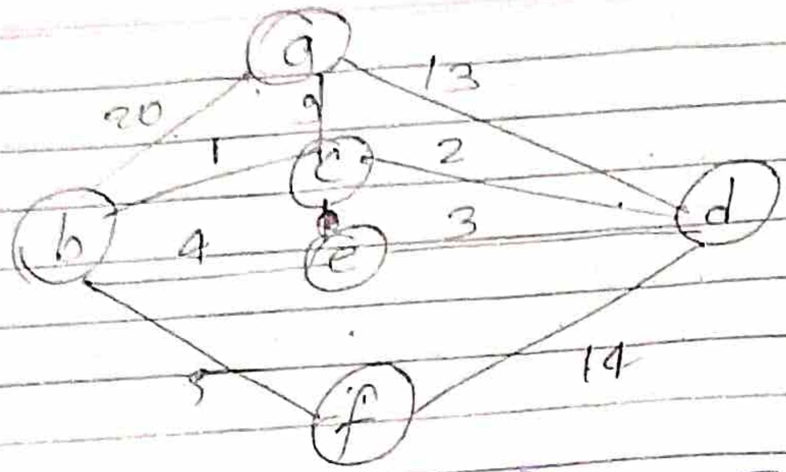
Step2: Select an edge that connects the tree with a vertex not yet in the tree so that the weight of the edge is minimal and inclusion of the edge does not form a cycle

Step3: Add the selected edge and the vertex that it connects to the tree

Step4: Repeat steps two and three untill all the vertices are included in the tree.

remmove loop and multiedge first

Q



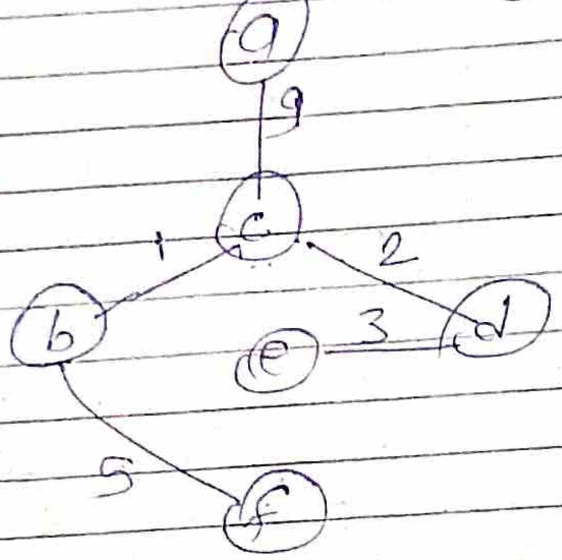
(a)

20 (9) 13

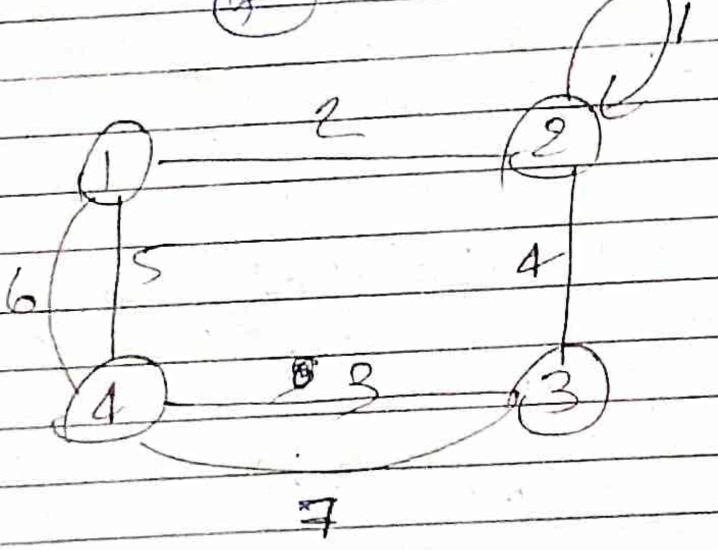
(1) (2)

4 5

(3) 14



Q.



Discrete Mathematics

MST-2 Solve {shared resources}

Q1. Construct all the possible spanning trees of K4 complete graph.

Sol^n: Total spanning tree of K4 complete graph is:

$$n^{n-2} = 4^{4-2} = 4^2 = \underline{16}.$$

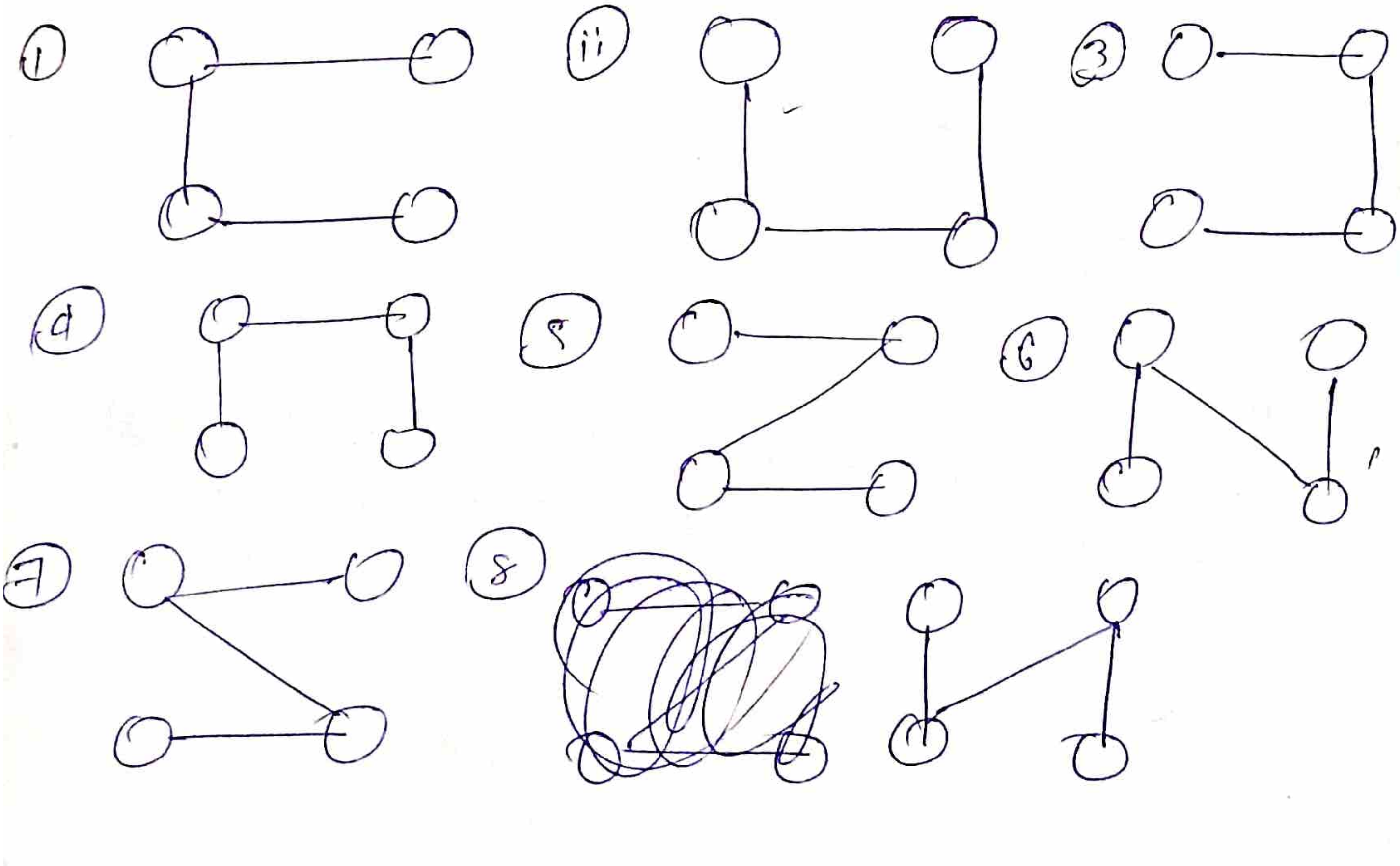
K4 mean no of vertices will be 4

so by using spanning tree property

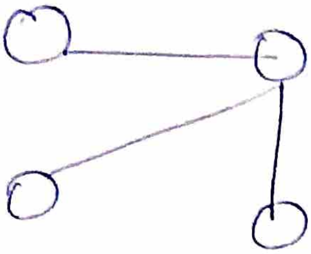
(i) A connected sub-graph S of a graph G(V, E), will be ST if all the vertices of the graph G S should contain.

(ii) S should contain (|V|-1) edges.

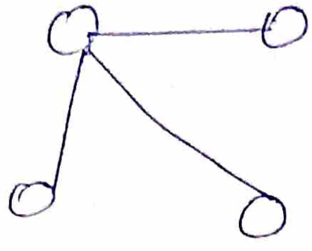
vertices = 4 Edges = 3



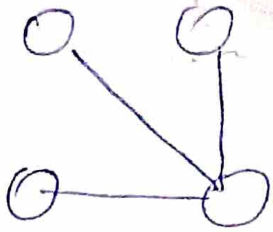
(9)



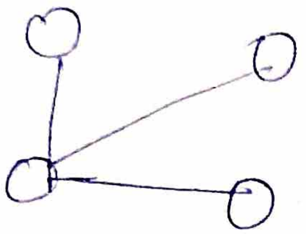
(10)



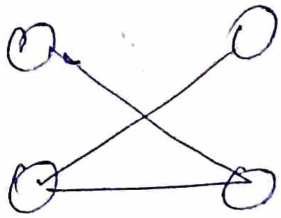
(11)



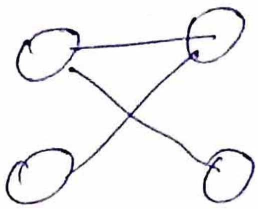
(12)



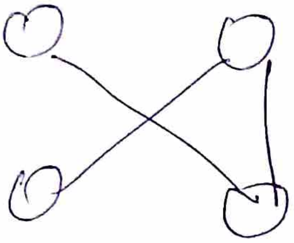
(13)



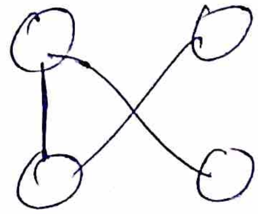
(14)



(15)



(16)



Q. Define the following terms with example

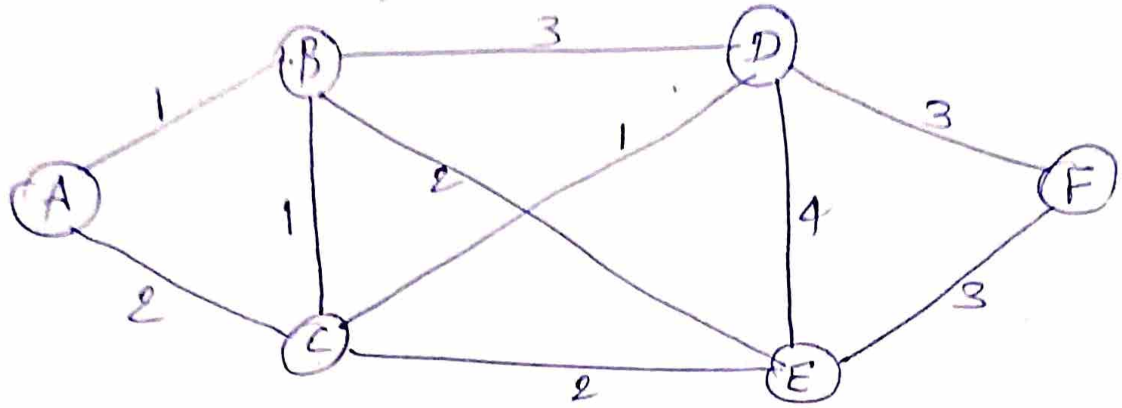
(a) Bipartite Graph ✓
(b) Hamiltonian Graph ✓
(c) chromatic number ✓
(d) Rings ✗
(e) Spanning Trees ✓

(a) Bipartite Same →

(PYQ) Compare and contrast the minimum spanning tree algorithms Prim's and Kruskal's algorithm.

Features	Prim's Algorithm	Kruskal's Algorithm
Approach	It is vertex based Algorithm. It grows the MST one vertex at a time.	It is Edge-based Algorithm. It adds edges in increasing order of weight.
Data structure	It uses priority queue (min heap)	union-find data structure.
Graph Representation	Adjacency Matrix or adjacency list.	Edge list
Edge Selection	chooses the minimum weight edge from the connected vertices.	chooses the minimum weight edge from all edges.
Suitable for	Dense Graphs	Sparse Graphs
Starting point	Requires a starting vertex.	No specific starting point, operates on global edges
Examples	Network designs	Road networks
Memory usage	more memory for priority queue	less memory if edges can be sorted.
Complexity	Relatively simpler in dense graphs	more complex due to cycle management.

Q Solve the following graph using floyd warshall's algorithm.



$D_0 =$

	A	B	C	D	E
A	0	1	2	∞	∞
B	1	0	1	3	2
C	2	1	0	1	2
D	∞	3	1	0	4
E	∞	2	2	4	0

$DA =$

	A	B	C	D	E
A	0	1	2	∞	∞
B	1	0	1	3	2
C	2	1	0	1	2
D	∞	3	1	0	4
E	∞	2	2	4	0

Interm Node = A

$DB =$

	A	B	C	D	E
A	0	1	2	4	3
B	1	0	1	3	2
C	2	1	0	1	2
D	4	3	1	0	4
E	3	2	2	4	0

Ind Node = A & B both

$DC =$

	A	B	C	D	E
A	0	1	2	3	3
B	1	0	1	2	2
C	2	1	0	1	2
D	3	2	1	0	3
E	3	2	2	3	0

Ind Nod = A, B, C
Com A, B & C

DD =

	A	B	C	D	E
A	0	1	2	3	3
B	1	0	1	2	2
C	2	1	0	1	2
D	3	2	1	0	3
E	3	2	2	3	0

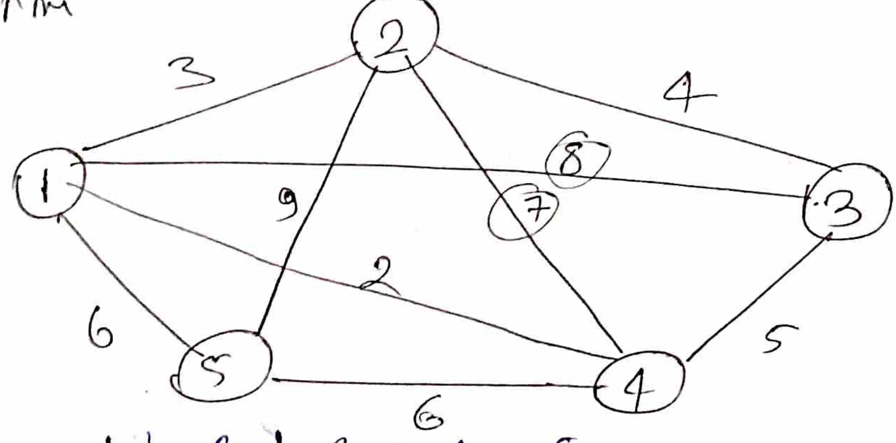
IN = A, B, C, D
com, A, B, C, D

DE =

	A	B	C	D	E
A	0	1	2	3	3
B	1	0	1	2	2
C	2	1	0	1	2
D	3	2	1	0	3
E	3	2	2	3	0

IN = A, B, C, D, E
com, A, B, C, D, E

Q. Solve the following graph ~~k~~ ~~us~~ using warshall's Algorithm



Do =

	1	2	3	4	5
1	0	3	8	2	6
2	3	0	4	7	9
3	8	4	0	5	∞
4	2	7	5	0	6
5	6	9	∞	6	0

~~IN =~~

$A =$

	1	2	3	4	5
1	0	3	8	2	6
2	3	0	4	5	9
3	8	4	0	5	14
4	2	5	5	0	6
5	6	9	14	6	0

IN = 1

$D_2 =$

	1	2	3	4	5
1	0	3	7	2	6
2	3	0	4	5	9
3	7	4	0	5	13
4	2	5	5	0	6
5	6	9	13	6	0

IN = 1 & 2
C = 1 & 2

$D_3 =$

	1	2	3	4	5
1	0	3	7	2	6
2	3	0	4	5	9
3	7	4	0	5	13
4	2	5	5	0	6
5	6	9	13	6	0

IN = 1, 2, 3
C = 1, 2, 3

$D_4 =$

	1	2	3	4	5
1	0	3	7	2	6
2	3	0	4	5	9
3	7	4	0	5	11
4	2	5	5	0	6
5	6	9	11	6	0

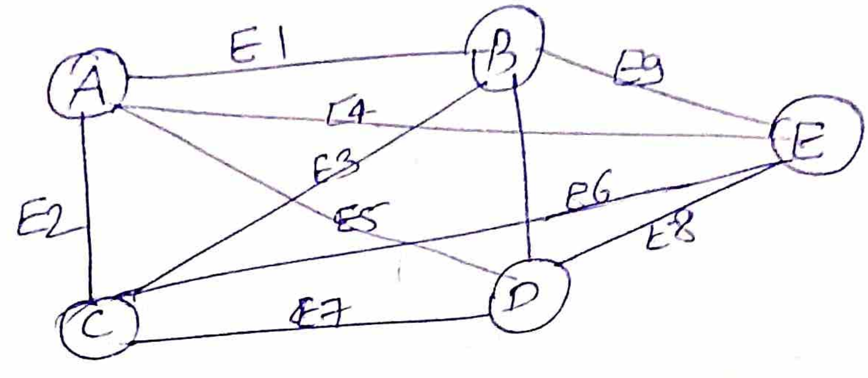
IN = 1, 2, 3, 4
Comm = 1, 2, 3, 4

$D_5 =$

	1	2	3	4	5
1	0	3	7	2	6
2	3	0	4	5	9
3	7	4	0	5	11
4	2	5	5	0	6
5	6	9	11	6	0

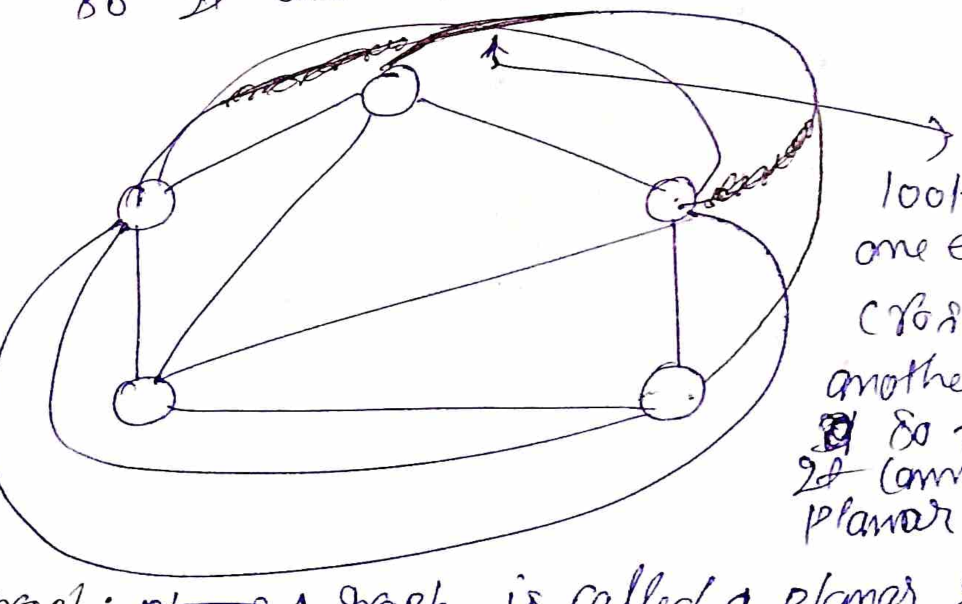
IN ⇒ 1, 2, 3, 4, 5
Comm ⇒ 1, 2, 3, 4, 5

19 Q. Construct a graph which has all the vertices of even degree but is not an Euler Circuit.



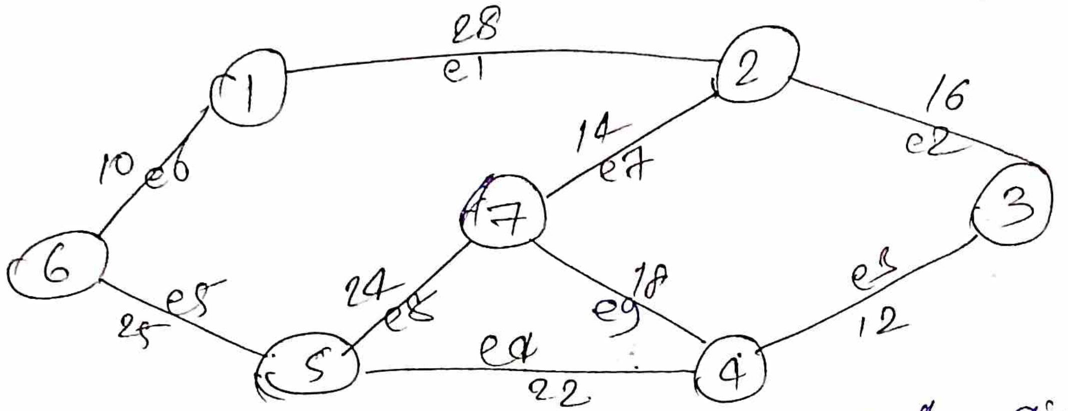
Q Show that a completegraph with 5 vertices is not a planar graph.

ans :) We know that from property of plannar graph that "A complete graph Kn is planar if and only if no of vertices (n) < 5".
Since now in this question no of vertices is 5 so It will not be a planar graph.



planar graph: A graph is called a planar graph if it can be drawn without any crossing edge.

(PYQS) construct a minimum spanning tree for the following graph using kruskal's algorithm. Also mention the weight of the same.



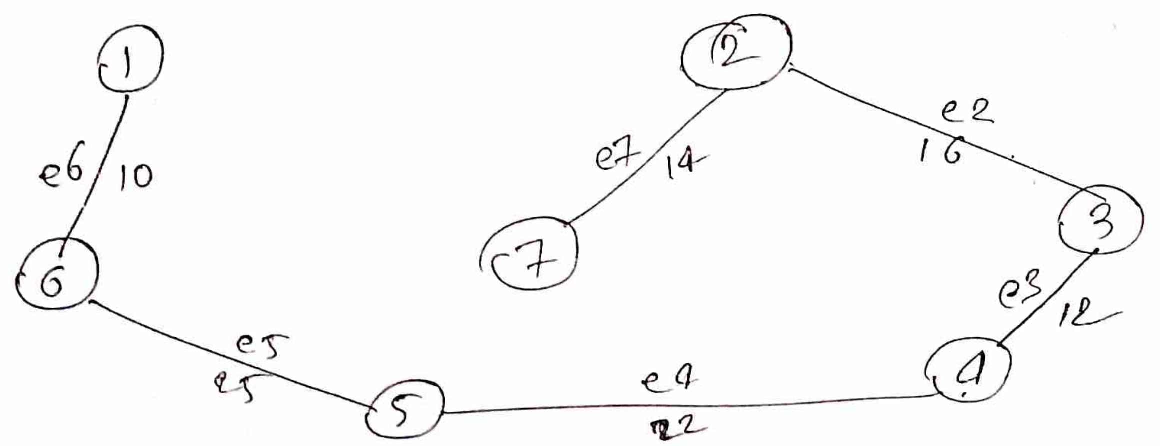
Step 1: Arrange all the edges of the given graph G(V,E) in ascending order. as per their edge weight

edges	vertices	edge weight	
e1	(1,2)	28	9
e2	(2,3)	16	4
e3	(3,4)	12	2
e4	(4,5)	22	6
e5	(5,6)	25	8
e6	(1,6)	10	1
e7	(2,7)	14	3
e8	(5,7)	24	7
e9	(4,7)	18	5

Ascending order

edges	vertices	edge weight
e6	(1,6)	10 ✓
e3	(3,4)	12 ✓
e7	(2,7)	14 ✓
e2	(2,3)	16 ✓
e9	(4,7)	18 ✗
e4	(4,5)	22 ✓
e7	(5,7)	24 ✗
e5	(5,6)	25 ✓
e1	(1,2)	28 ✗

Select ~~mini~~ smallest edge weight and form graph, if cycle is there then discard it



Minimum Spanning Tree ⇒

Total edge weight $= 14 + 16 + 12 + 22 + 25 + 10$

$= 99$

final Questions

Q. prove that following is a contingent:

$(q \wedge p) \vee (q \wedge \sim p)$

(1)

Q. State pigeonhole principle with example.

ans ⇒ If n pigeons are assigned to m pigeonholes and if $(n \geq m)$, then at least one pigeonhole contains two or more pigeons.

Example: suppose there are 11 pigeons and 10 pigeonholes. According to the pigeonhole principle, if each pigeon is placed in one of the pigeonholes, at least one pigeonhole will contains more than one pigeon.

$n \geq m$

pigeons $(n) = 11$, Pigeonholes $= 10$ $(n > m)$

if each pigeons is placed in a pigeonhole, at least one of the pigeonholes must contains more than one pigeon, because there are more pigeons than pigeonshole

Chapter - Algebric structure/system

① **closuer property:** A non empty set S is called algebric structure or system with respect to binary operation * if $(a*b) \in S \ \forall (a,b) \in S$.

Ex ⇒ $S=\{N,+\}$, $5+10=15 \in S$ | $S=\{N,\cdot\}$ (multiplication), $5\times 1=5 \in S$ | $S=\{Q^*,/\}$ (division)

- → Q → rational number
  Ex ⇒ $\frac{1}{2}, \frac{3}{1}, \frac{1}{1}, 5, 6$
- → Q^* → non-zero rational number
  Ex ⇒ $\frac{1}{2}, \frac{3}{1}, \frac{1}{1}, 5, 6$ not 0
- → Tx → irrational number
- → R → real number
  Ex ⇒ (−ve, 0, +ve)
- → C → complex No
  Ex ⇒ $z=a+ib$ ⇒ $3+4i$
- → N → Natural No
  Ex → $\{1,2,3,4,\ldots\}$
- Z → Integer (+ve, −ve)

② **Semigroup:** An algebric structure $(S,*)$ is called a semigroup if it follows ~~allociatede~~ associative property.

(Semigroup = closuer + Associative property)

$$(a*b)*c = a*(b*c) \ \forall (a,b) \in S$$

Ex ⇒ (i) $S=(N,+)$

$S=2+2=4 \in S$

$(1+2)+3 = 1+(2+3)$

$6=6$ semigroup

(ii) $S=(Z,*) \Rightarrow (1*2=2) \Rightarrow 1*(2*3)=(1*2)*3 \Rightarrow 6=6$

③ **Monoid:** A semigroup is called monoid if there exists an ~~element~~ identity element e in ~~set~~ S such that

$$\boxed{a*e=e*a=a} \ \forall (a \in S)$$

identity element

Ex ⇒ (i) $S=\{N,+\}$

$5+\underline{0}=5$

$\underline{0}+5=5$

(ii) $S=\{Z,*\}$

$2*\underline{1}=2$

$\underline{1}*2=2$

(4) Group: A non empty set S is called a group if it satisfies following property.

(i) closure property $\rightarrow$ $5 \in N, 6 \in N, 5+6=11 \in N$
$(a*b) \in S \; \forall (a,b) \in S$

(ii) Semigroup $\rightarrow$ $a*(b*c) = (a*b)*c \; \forall (a,b,c) \in S$

(iii) Monoid $\rightarrow$ identity ele $\rightarrow$ $a*e = a = e*a = a \quad \forall (a \in S)$

(iv) Existence of Inverse: Each element of set S is invertable if there exist a^{-1} in $S \; \forall (a \in S)$ such that

$$\boxed{a*a^{-1} = a^{-1}*a = e} \; \forall (a \in S)$$

$e \rightarrow$ identity element

Ex $\Rightarrow$ (i) $S = (\mathbb{Z}, +) \Rightarrow$

$5 + (-5) = 0 (e)$
$(-5) + 5 = 0 (e)$

e in + = 0
e in · = 1

(ii) $S = \{R^*, \cdot\} \Rightarrow 5 * \frac{1}{5} = \underset{(e)}{1}$ या $\frac{1}{5} \times 5 = 1 (e)$

(5) Abelian Group: A Group $(G, *)$ is said to be abelian if it satisfies commutative property.

$$(a*b) = (b*a) \quad \forall (a,b \in G)$$

Ex $\Rightarrow$
(i) $G = (Z, +) \Rightarrow 10+7 = 7+10$ ✓
(ii) $G = \{R^*, \cdot\} \Rightarrow 5 \times 5 = 5 \times 5$ ✓

(6) Rings: A abelian Group $(G, *)$ is said to be Rings if it satisfies Distributive laws.

$$\boxed{a \cdot (b+c) = a \cdot b + a \cdot c} \; \forall (a,b,c \in G)$$

Ex $\Rightarrow$ (i) $S = \{Z, +, \cdot\}$

$2 \cdot (3+4) = 2 \cdot 3 + 2 \cdot 4$
$2 \cdot 7 = 6 + 8$
$14 = 14$ ✓

Q. Consider the Group $G = \{1,2,3,4,5,6\}$ under the multiplication modulo 7. prove that G is a group.

Sol: For G to be a group, we need a binary operation let's consider the operation to be multiplication modulo 7. (• mod 7)

let Multiplication Modulo = 7

$\boxed{X_7 \to MM}$ (mul, %)

$\boxed{((X_m) * (X_n)) \% 7}$

composition table

X_7	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

(i) closure property: we can see that all the entries in composition table are the elements of set G ∴ G is closure w.r.t X_7. look → $(a*b) \in G$ means $(a X_7 b) \Rightarrow (2 X_7 4) = 1 \in G$

(ii) Semigroup (Associative): We know that the composition table are always associative ∴ G is semigroup w.r.t X_7.

$(a \cdot b) \cdot c \mod 7 = a \cdot (b \cdot c) \mod 7$

$(1 \cdot 2) \cdot 3 \mod 7 = 1 \cdot (2 \cdot 3) \mod 7$

$6 \mod 7 = 6 \mod 7$

$6 = 6$ ✓

$(a X_7 b) X_7 c = a X_7 (b X_7 c)$

$(2) X_7 (4)) X_7 (5) = (2) X_7 (4 X_7 5)$

$(1) X_7 (5) = 5 = (2) X_7 (6)$

look $= 5$

(iii) Monoid (identity pro): It follows identity property

$\boxed{a X_7 e = e X_7 a = a}$

~~$X_7 1 = 2 \mod 7 \ldots$~~ ∴ G is Monoid w.r.to X_7.

~~$X_7 2 = 2 \mod 7$~~ $\boxed{e = 1}$

$(4) X_7 (1) = 1 (X_7) 4$

$1a = a = a$

(iv) Group (Existance of inverse): For each $\forall (a,b \in G)$ Such that: $(a \times_7 a^{-1} = a^{-1} \times_7 a = e)$

$a \cdot b = 1 \mod 7$

$1 \cdot 1 = 1 \mod 7 = 1$ so the inverse of 1 is 1.

$2 \cdot 4 = 8 \mod 7 = 1$ inverse of 2 is 4

$6 \cdot 6 = 36 \mod 7 = 1$ inverse of 6 is 6.

each ele has an inverse in G.

$\therefore$ G is a Group w.r.t $\times_7$.

(v) Abelian Group (Commutative law): for $\forall (a,b \in G)$ such that

$(a \times_7 b) = (b \times_7 a)$ $\quad 2 \times_7 4 = 1$ या $(4) \times_7 (2) = 1$

Ex) ~~2·3 = (2×3) mod 7 = 6 mod 7 = 6~~ it follows comm law

~~3·2 = (3×2) mod 7 = 6 mod 7 = 6~~ with r. to $\times_7$.

(vi) Explain Isomorphic Graph: Any two graphs will be isomorphic graph if they satisfy the following four conditions.

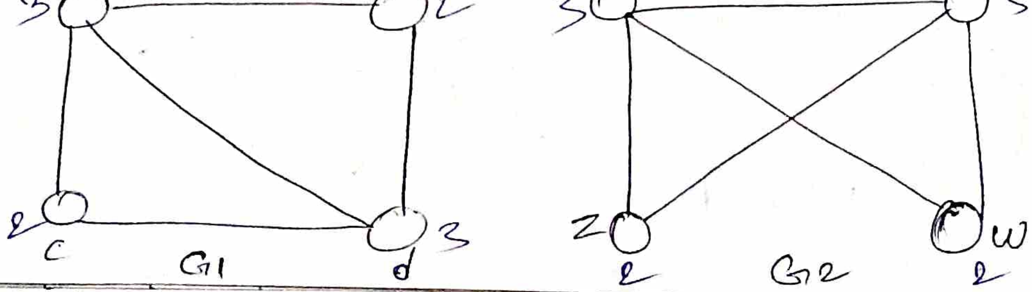
(i) There should be an equal no of vertices in the given graphs.

(ii) There will be an equal no of edges in the given graph.

(iii) There will be an equal amount of degree sequence in the given graph.

(iv) vertex correspondance & edge correspondance valid

Ex ⇒

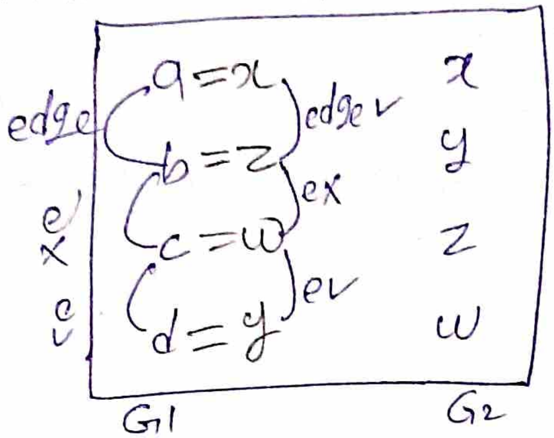


Condition 1 : No of vertex : $G1 = 4$, $G2 = 4$

" 2 : No of edges : $G1 = 5$, $G2 = 5$

" 3 : degree sequence : $G_1 = 3\ 3\ 2\ 2$, $G2 = 3\ 3\ 2\ 2$

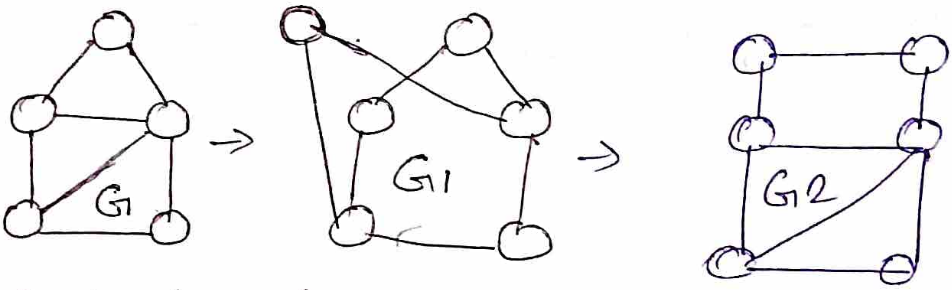
4 : mapping (vertex correspondance & edge correspondance)



both graphs are isomorphic

(PYQ) Homeomorphic graph : Two graphs G_1 & G2 are said to be Homeomorphic graph if and only if each can be obtained from a graph G by adding vertices to edges.

EX ⇒



(PYQ) what is the application of kruskal's algorithm?

ans ⇒ kruskal's Algorithm is primarly used for finding the Minimum spanning Tree of a graph.

Applications.

(i) Network Design : It is used to design least cost networks such as computer networks, road networks, electrical circuit.

(ii) Transportation Network : In transportation, kruskal's algorithm can help in creating the most cost effective transport routes.

(iii) Telecommunication networks : Kruskal's algorithm helps design telecommunication networks by connecting different towers or switches to minimize total cable length.

(iv) computer graphics: It is used in the generation of spanning trees for image processing like in rendering 3D objects.

Q) If R is an equivalence relation, prove that inverse R^{-1} is also an equivalence relation.

ans ⇒ Since R is equivalence relation on a set A

$\therefore$ R is reflexive, symmetric and transitive

Now R is reflexive $\Rightarrow (a,a) \in R \ \forall a \in A$

$(a,a) \in R^{-1} \ \forall a \in A$

Thus R^{-1} is reflexive

(ii) Transitive ~~Symmetric~~: Let $(a,b) \in R^{-1}$ and $(b,c) \in R^{-1}$

Now $(a,b) \in R^{-1}$ and $(b,c) \in R^{-1}$

$(b,a) \in R$ and $(c,b) \in R$

$(c,b) \in R$ and $(b,a) \in R$

$\{\because R$ is transitive$\}$

$(c,a) \in R$

$(a,c) \in R^{-1}$ Thus R^{-1} is transitive.

(iii) Symmetric: Let $(a,b) \in R^{-1}$

Now $(a,b) \in R^{-1} \Rightarrow (b,a) \in R$ ($\because R$ is symmetric

$(a,b) \in R$

$(b,a) \in R^{-1}$

Thus R^{-1} is symmetric

PYQ) prove that in any graph, there is even number of vertices of odd degree.

Solⁿ: Let $G = (V, E)$ is a undirected graph.
Let U the set of even degree vertices in G
and let W the set of odd degree vertices in G.

Then,

$$\sum_{v_i \in V} deg(v_i) = \sum_{v_i \in U} deg(v_i) + \sum_{v_i \in W} deg(v_i)$$

by handshaking lemma $\Rightarrow \boxed{\sum_{v_i \in V} deg(v) = 2e}$

$$\underbrace{2e}_{even} - \underbrace{\sum_{v_i \in U} deg(v_i)}_{even} = \sum_{v_i \in W} deg(v_i) \quad \text{— (1)}$$

even − even = even

Now, $\sum_{v_i \in W} deg(v_i)$ is also even, therefor, from eq (1)

$\boxed{\sum_{v_i \in W} deg(v_i) \text{ is even}}$ ∴ the no of odd vertices in G is even

PYQ) Differentiate between inclusion and exclusion principle.
The inclusion-exclusion principle is a combinatorial method used to calculate the size of the union of multiple sets. It helps to avoid over-counting elements that may belong to more than one set.

Inclusion principle: The inclusion principle refers to the concept of counting all elements that belong to at least one of the sets. It emphasizes the idea of including ~~element~~ elements from multiple sets in a total count.

Ex ⇒ If you have two sets A and B, their union

$\boxed{n(A \cup B) = n(A) + n(B)}$ $\quad$ $A = \{1, 2, 3, 4\}$, $B = \{5, 6, 7, 8\}$

or $\boxed{n(A \cup B) = \{1, 2, 3, 4, 5, 6, 7, 8\}}$ $\quad$ $\boxed{n(A \cup B) = 4 + 4 = 8}$

(ii) Exclusion principle: The exclusion principle addresses the issue of over-counting elements that belongs to more than one set. It involves removing the counts of elements that have been included multiple times due to overlaps between sets.

Ex ⇒ If you have two sets A and B.

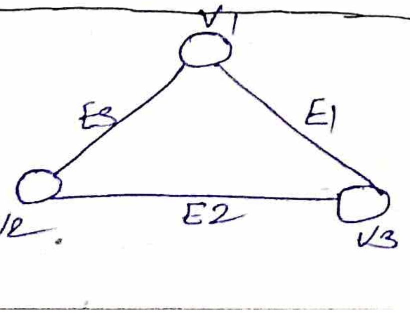
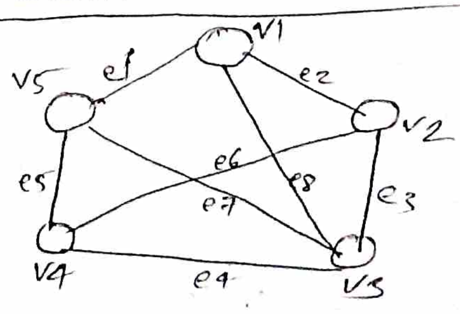
formula ⇒ $\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$

$A = \{1, 3, 5, 2, 7, 4\}$, $B = \{3, 5, 2, 7, 8, 6\}$

$n(A \cup B) = 6 + 6 - (4) \Rightarrow 12 - 4 = 8$

$\boxed{n(A \cup B) = 8}$

PYQ what is difference b/w a cycle and Hamiltonian cycle.

features	cycle	Hamiltonian cycle
Definition	A closed path in a graph where the starting and ending vertex are the same.	A closed path in a graph that visits every vertex exactly once and returns to the starting vertex.
vertex coverage	Does not necessarily include all vertices of the graph.	Includes all vertices of the graph exactly once.
Graph requirements	Can exists in any Graph.	Requires a connected graph
use	It is used in various graph structures.	Used in problems like the Travelling salesman problem.
Example:		

Q which is better Krufkal or prims algorithm?

ans => which is better algorithm depends on the specific characteristics of the graph.

Kruskal's Algo: strength

(i) Efficient for sparse graphs

(ii) simple to implement

(iii) can handle disconnected graphs

weaknesses:

(i) Requires sorting of all edges (time consuming)

(ii) uses a disjoint-set data structure.

prim's Algo: strength

(i) Efficient for dense graphs

(ii) often faster than kruskal's for dense graphs.

(iii) It uses a priority queue data structure:

weakness:

(i) can less efficient to implement than kruskal's

(ii) Requires the graph to be connected.

- Sparse Graphs: kruskal's algo is generally preferred.
- Dense Graphs: prim's algo is ~~often~~ generally preferred.
- Disconnected Graphs: kruskal's Algo is the only option.

In conclusion, there's no definitive answer to which algo is better. The best choice depends on the specific graph structure.

PYQ Compare between floyd warshall and Kruskal Algorithm :

Features	Floyd-warshall Algo	Kruskal's Algorithm
Definition	It finds shortest path between all pairs of vertices in a graph.	It finds the minimum spanning tree with minimum no of edge weight.
Graph type	weighted, directed or undirected graph.	weighted, undirected graphs.
Approach	Dynamic programming	Greedy Algorithm.
Time complexity	$O(V^3)$ V → vertices	$O(E \log V)$ V → vertices, E → edges
Space Complexity	$O(V^2)$	$O(V+E)$
Negative weights	Handles negative edge weight.	Does not handle negative edge weights
Use cases	Suitable for dense graph	suitable for sparse graph
update to graph	Easy to update	Difficult to update

PYQ Explain the various Applications of Graph.

(i) Computer Networks: Graph represent computer networks where nodes are devices and edges represent connections.

(ii) Social Networks: Social networks like facebook or LinkedIn can be represented using graphs, where nodes represent user and edges represent relationships.

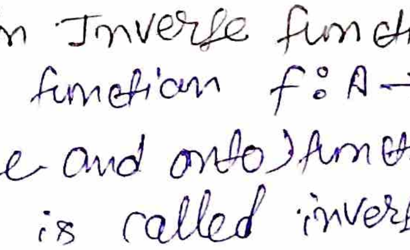
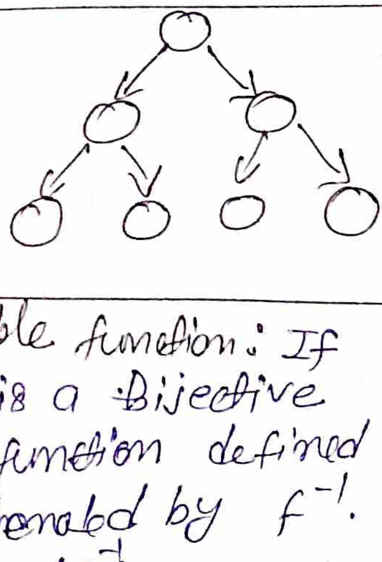
(iii) Web page Ranking: The world wide web is modeled as a graph, where web pages are nodes and hyperlinks are edges. Google's PageRank algorithm uses graphs to rank w..

(iv) Transportation and Navigation: Graph model transportation systems where nodes are locations and edges are routes.

(v) Electrical circuits: Graph model electrical circuits where nodes are components and edges represent connections.

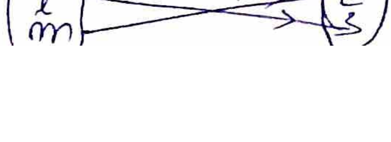
(vi) Supply chain and Logistics: Graph represent supply chains, where nodes are suppliers, manufacturers, and distributors. and edges are transportation routes.

(PYQ) Diff b/w Graph and Tree

Features	Graph	Tree
Definition	A collection of nodes (vertices) connected by edges.	A hierarchical structure consisting of nodes, where each nodes has one parent. except root node.
Root Node	No root Node	Has a unique root node
cycle	can have cycles	Does not contain cycle.
connection less	can be connected or disconnected	Must be connected.
path	Multiple paths can exist	Exactly one unique path exits.
Example		

(PYQ) Explain Inverse function / Invertible function: If f is a function $f: A \to B$ and is a Bijective (one-one and onto) function, Then function defined by $g: B \to A$ is called inverse of f denoted by f^{-1}.

Ex: $f: x \to y$ $f = \{(1,k)(2,m)(3,l)\}$



PYQ) diff b/w equal and equivalent set.

features	Equal set	Equivalent set
Definition	Sets that contain exactly the same elements	Sets that have the same no of elements
Elements	Must be identical in both sets.	It can have different elements.
cardinality	Same number of elements	Must have the same number of elements
~~Stand~~ Example	$A=\{1,2,3\}$ $B=\{1,2,3\}$	$A=\{1,2,3\}$ $B=\{a,b,c\}$

PY 8) State and prove Euler's theorem.

Euler's formula: let $G(V,E)$ be a connected planer simple graph and R be the no of regions in planer representation of G, then

$$\boxed{R = E - V + 2}$$

$V \rightarrow$ vertices, $E \rightarrow$ edges
$R \rightarrow$ Regions

<u>proof</u>

(i) Base $\quad n=1$

$$r_1 = e_1 - v_1 + 2$$
$$1 = 1 - \cancel{2} + \cancel{2}$$
$$1 = 1 \quad \text{Result is true for } n=1$$

(2) Induction step: Assume $\boxed{n=k}$

$$r_k = e_k - v_k + 2 \quad \text{—— (2)}$$

(3) verification step: $\boxed{r_{k+1} = e_{k+1} - v_{k+1} + 2}$ —— (3)

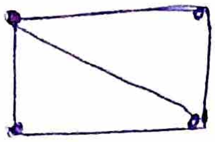
induction step →

$n = K$

from eq (2)

$G_K = (V_K, e_K, r_K)$

$n = K+1$



$e_K = e_{K+1} - 1$

$V_K = V_{K+1}$

$r_K = r_{K+1} - 1$

$G_{K+1}(V_{K+1}, e_{K+1}, r_{K+1})$

put in eq (2)

$r_K = e_K - V_K + 2$

$r_{K+1} - \cancel{1} = e_{K+1} - \cancel{1} - V_{K+1} + 2$

$\boxed{r_{K+1} = e_{K+1} - V_{K+1} + 2} = \text{eq } (3)\ \boxed{r_{K+1} = e_{K+1} - V_{K+1} + 2}$

hence proved

(PYQ) Determine the minimum no of people to have guarantee that at least two of them have birthday that occur on same day of the week.

ans → We can slove this using the pigeonhole principle

There are 7 days in a week

so minimum no of people should be 8 so that at least two people must share a birthday on the same day of the week.

so Pigeonhole = 7 (week)

pigeon = 8

$\boxed{\text{People} = 8}$

P1	P2, P8	P3	P4	P5	P6	P7
S	M	T	W	T	F	S

↓ at least 2 person P2, P8 have birthday on the same day.

(Q15) Justify the significance of hash function in computer science.

ans ⇒ Significance of hash function.

① Data Integrity: Hash functions are used to verify the integrity of data. By generating a hash value for a data. any changes to that data will result in a different hash value.

② Data structure: Hash functions are widely used in data structures for efficient data retrieval. By mapping key to a specific locations in an array.

③ Deduplication: In storage system, hash functions are used to identify duplicate files or data. By hashing files and comparing their hash values.

④ Blockchain: Hash functions are fundational to blockchain technology. They ensure the integrity and immutability of the blockchain by linking blocks of transactions through cryptographic hashes.

⑤ Performance and Efficiency: Hash functions are designed to be fast and efficient, allowing for quick computation of hash values.

Propositional and predicate logic

* **Propositional or Sentence:** An expression consisting of some symbols, letters and words is called propositional or sentence if it is true or false.

example: Jaipur is capital of Rajasthan True

$2+3=5$ True

$9<6$ false

True value: If any proposition is true than its truth value is denoted by T and if the proposition is false then its truth value is denoted by F.

Ex ⇒ 1 is less than 3 T

14 is odd no F

* **Types of proposition**

(i) simple proposition: The proposition having one subject and one predicate is called a simple proposition.

Ex ⇒ (i) This flower is pink.

(ii) Every even number is divisible by 2.

(ii) Compound proposition: Two or more simple proposition when combined by various connectivities into a single composite sentence is called compound proposition.

Ex ⇒ (i) The earth is round and revolves around the sun.

(ii) A triangle is equilateral iff its three sides are equal.

* **Logical Connectives:** The particular words and symbols used to join two or more proposition into a single composite form or compound proposition are called logical connectives.

Logical connectives words	symbol	uses
And/conjuction/join	$\wedge$	$p \wedge q$
or/disjunction/meet	$\vee$	$p \vee q$
Negation	$-$ or $\sim$	$\sim p$
Equivalent	$\leftrightarrow$	$p \leftrightarrow q$
Conditional "if....then..."	$\Rightarrow$	$p \Rightarrow q$
Biconditional "if and only if" (iff)	$\Leftrightarrow$	$p \Leftrightarrow q$
NAND (NOT + AND)	$\uparrow$	$p \uparrow q$
NOR (NOT + or)	$\downarrow$	$p \downarrow q$
XOR	$\oplus$	$p \oplus q$

Basic Logical operation pyq) what are conjunction and disjunction operations.

① Conjunction of: Any two proposition can be combined by the word "And" to form a compound proposition said to be the conjuction.

Truth table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Ex ⇒

(i) Delhi is in india and 2+2=4 (T)

(ii) Delhi is in india and 2+2=5 (F)

② Disjunction of: Any two-proposition can be combined by the word 'or' to form a compound proposition is said to be the disjuction.

Truth table

p	q	$p \vee q$
T	T	T
T	f	T
f	T	T
f	f	f

Example: (i) Delhi is in india or 2+2=4 T

(ii) Delhi is in india or 2+2=5 (T)

(iii) Delhi is in Russia or 2+2=6 (F)

(3) Negation: The negation proposition of any given proposition P is the proposition whose truth value is opposite to P.

P	~P
T	F
F	T

Ex→

(1) P ≡ "This flower is pink"

(2) ~P ≡ "This flower is not pink"

(4) Tuotologies and Contradiction: A proposition is said to be an tautologies if it contain only T in last column of truth table.

Contradiction: A proposition is said to be an contradiction if it containe only F in last column of truth table.

Contingent: If it contains both T and F.

Q1. Show that the following proposition is tautologies

$\{(P \vee \sim Q) \wedge (\sim P \vee \sim Q)\} \vee Q$

Solⁿ:

P	Q	~P	~Q	E = (P) ∨ (~Q)	G = (~P) ∨ (~Q)	F = E ∧ G	F ∨ Q
T	T	F	F	T	F	F	T
T	F	F	T	T	T	T	T
F	T	T	F	F	T	F	T
F	F	T	T	T	T	T	T

Yes this is tuotologies.

Note: if equivalent then two statement will be given like $P \vee \sim(q \wedge r)$ and $(P \vee \sim q) \vee \sim r$ are equivalent then

$\underbrace{P \vee \sim(q \wedge r)}_{C1}$ and $\underbrace{(P \vee \sim q) \vee \sim r}_{C2}$

find C1 and C2 if C1 last column and C2 last column are equal then it will be equivalent.

* Conditional statement: Many statements are of the form "if p then q" such statement are said to be the Conditional statement and denoted by $p \Rightarrow q$ or $p \rightarrow q$.

Truth Table

p	q	p→q
T	T	T
F	F	T
T	F	F
F	T	T

Note: how to solve table

$p \rightarrow Q$

(1) P→ मे आता - जहाँ false है वहाँ True o/p होगा

(2) अगर T है तो Q को देखो

$p \rightarrow T \quad Q \rightarrow T = T$

$p \rightarrow T \quad Q \rightarrow F = F$

* Biconditional statement: A statement "p if and only if q" such statement are said to be bi-conditional statement and denoted by $p \Leftrightarrow q$ or $p \leftrightarrow q$.

Truth table

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

diff → F

same → T

(PYQ) Show that $(P \rightarrow Q) \rightarrow [(P \rightarrow Q) \rightarrow Q]$ is tautology or not.

A = B =

P	Q	(P→Q)	[A→Q]	A→B
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	F

It is not tautology because all the values in last column is not true.

* Normal form

(i) Disconjunction Normal form (DNF): A statement form which consist of disjunction between conjunction is called (DNF).

Ex → ① $(p \wedge q) \vee r$ ② $(p \wedge \sim q) \vee (\sim p \wedge r) \vee (r \wedge \sim q)$

conjunction disjunction

properties

Some important law

(i) idempotent law → $p \wedge p \Leftrightarrow p$ and $p \vee p \Leftrightarrow p$

(ii) commutative law → $p \wedge q \Leftrightarrow q \wedge p$ and $p \vee q \Leftrightarrow q \vee p$

(iii) Associative law ⇒ $(p \vee q) \vee r = p \vee (q \vee r)$

(iv) De-Morgan law ⇒ $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$ and

$\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

distributive law

(v) $(\sim p \vee q) \wedge (\sim p \wedge q) = (\sim p \wedge \sim p \wedge q) \vee (q \wedge \sim p \wedge q)$

⑥ $\boxed{p \rightarrow q \Leftrightarrow \sim p \vee q}$ ⑦ $\boxed{(q \leftrightarrow p) \Rightarrow (q \rightarrow p) \wedge (p \rightarrow q)}$

Q. obtain the DNF of the form $(p \rightarrow q) \wedge (\sim p \wedge q)$

Solⁿ: we know that $p \rightarrow q \Leftrightarrow \sim p \vee q$

$(\sim p \vee q) \wedge (\sim p \wedge q)$ $\{(A \vee B) \wedge (B \wedge C) = (A \wedge (B \wedge C)) \vee (B \wedge (B \wedge C))$

Applying distributive law

$(\sim p \wedge \sim p \wedge q) \vee (q \wedge \sim p \wedge q)$

Apply idempotent law $p \wedge p \Leftrightarrow p$

$\boxed{(\sim p \wedge q) \vee (q \wedge \sim p)}$ DNF

* Conjunction Normal form (CNF): A statement form which consists of conjuction between disjunction is called CNF.

Example: (i) $p \wedge q$ (ii) $(\sim p \vee q) \wedge (\sim p \vee r)$

Q. obtain CNF of the form $(p \wedge q) \vee (\sim p \wedge q \wedge r)$

Solⁿ: using distributive law

$[p \vee (\sim p \wedge q \wedge r)] \wedge [q \vee (\sim p \wedge q \wedge r)]$

$[(p \vee \sim p) \wedge (p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge (q \vee q) \wedge (q \vee r)]$

$p \wedge \sim p = 1$ या $p \wedge \sim p = 1$, $q \vee q = q$ Idempotent

$\boxed{(p \vee q) \wedge (p \vee r) \wedge [(q \vee \sim p) \wedge q \wedge (q \vee r)]}$

CNF

Q obtain CNF of $(p \rightarrow q) \wedge (q \vee (p \wedge r))$

Solⁿ: we know that $(p \rightarrow q) \Rightarrow (\sim p \vee q)$

$(\sim p \vee q) \wedge (q \vee (p \wedge r))$

$(\sim p \vee q) \wedge ((q \vee p) \wedge (q \vee r))$

$\boxed{(\sim p \vee q) \wedge (q \vee p) \wedge (q \vee r)}$ CNF

(PYQ) obtain the CNF of the form $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$.

Sol: we know that $p \leftrightarrow q \Rightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

we also know that $p \rightarrow r \Rightarrow (\sim p \vee r)$

$(\sim(\sim p) \vee r) \wedge (p \rightarrow q) \wedge (q \rightarrow p)$

$\boxed{(p \vee r) \wedge (\sim p \vee q) \wedge (\sim q \vee p)}$

CNF

(PYQ) prove that following is a contigent.

$(q \wedge p) \vee (q \wedge \sim p)$

Sol^n:

p	q	$\sim p$	$q \wedge p$ = A	$(q \wedge \sim p)$ = B	$A \vee B$
T	T	F	T	F	T
T	f	F	F	F	F
f	T	T	F	T	T
f	f	T	f	F	F

Since the statement is true for some combinations and false for others so it is 'contingent'.

* Validity of Well-formed formulas (WFFs)

In propositional logic, a well-formed formula (WFF) is a syntatically correct expression constructed using logical connectives and propositional variables.

(i) Validity: A WFF is valid if it is a tautology.

(ii) Satisfiability: A WFF is satisfiable if it is contingent.

(iii) Unsatisfiability: A WFF is unsatisfiability if it is a contradiction.

(PYQ) Consider following well-formed formulas in prepositional logic (a) $p \to p'$ (b) $(p \to p') \vee (p' \to p)$ which of these is valid and not valid

Sol^n:

p	p'	$p \to p'$ = A	$p' \to p$ = B	$A \vee B$	
T	F	F	T	T	
T	F	F	T	T	
F	T	T	f	T	
f	T	T	f	T	

(a) $p \to p'$ is not tautology so that it is not valid

(b) It is tautology so that it is valid

Q.15) write following statement in symbolic form:
if Kevin is not in a good mood or he is not busy, then he will go to Mumbai.

ans → Let P: Kevin is in a good mood

Q: Kevin is busy

R: Kevin will go to Mumbai

$\boxed{(\sim P \vee \sim Q) \rightarrow R}$ ans

* predicate Logic: It is a sentence that contains a finite number of variables. It becomes a proposition when specific values are substituted for the variables.

Ex → Ram is a teacher.

$P(x)$: x is a bachelor.

x → predicate variable, $P(x)$ → propositional function

Ex → $P(x): x > 3$ what are the truth values of $P(4)$ & $P(2)$.

$P(4): 4 > 3$; which true

$P(2): 2 > 3$ false

$P(x, y): x = y + 3$ at $P(3, 0)$

$P(3, 0): 3 = 0 + 3$ true

* Quantifiers: Quantifiers are words that refer to quantities such as 'some', 'few', 'many', 'all', 'none' and indicate how frequently a certain statement is true.

Types of [redacted] quantifiers

① universal quantifier ② Existential quantifier

universal quantifier	existential quantifier
1. The phrase "for all" is called universal quantifier.	The phrase "there exists" is called existential quantifier.
2. It is denoted by $\forall$.	It is denoted by $\exists$.
3. $\forall x$ represents "for all x" "for every x" "for each x"	$\exists x$ represents "there exists an x" "there is an x" "for some x" "there is at least one x"
4. All human beings are mortal $H(x)$: x is human $M(x)$: x is mortal $\boxed{\forall x\,(H(x) \rightarrow M(x))}$	4. Some human beings are mortal $H(x)$: x is human $m(x)$: x is mortal $\boxed{\exists x\,(H(x) \wedge M(x))}$

PYQ Compare between propositional and predicate logic

features	propositional logic	predicate logic
Definition	Deals with simple propositions that are either true or false.	Extends propositional logic by including quantifiers and variables.
Components	connectives ($\wedge, \vee, \sim$, etc)	variables, predicate, quantifiers ($\forall$, $\exists$).
variables	Not present	uses variables.
quantifiers	Not supported.	supports
Scope	Limited to specific proposition	Broader scope
complexity	Simpler.	complex
Expressiveness	Less Expressive.	More expressive
Example	"It is raining" (Proposition P).	"All humans are mortal" $\boxed{\forall x\,(H(x) \rightarrow M(x))}$