Mathematics-II

BSC - 104

Fourier Series

Syllbus ⬇

⇒ fourier Series ÷ Periodic function, even & odd function, Euler's formula for fourier series, Dirichlet's condition, half range fourier series.

* Requarement topic for this chapter ÷

(i) odd & even function
(ii) Properties of Definite integrals

(I) $f(-x) = f(x)$ even function
$f(-x) = -f(x)$ odd function

like ÷ $f(\sin(-x)) = -\sin x$ (it is odd function)
$f(\cos(-x)) = \cos x$ (it is even function)

(II) $\sin(-x) = -\sin x$
$\cos(-x) = \cos x$
$\tan(-x) = -\tan x$
$\operatorname{cosec}(-x) = -\operatorname{cosec} x$
$\sec(-x) = \sec x$
$\cot(-x) = -\cot x$

(III) $\boxed{\sin(n\pi) = 0} \Rightarrow \sin\pi, \sin 2\pi, \sin 3\pi \ldots \Rightarrow 0$

$\boxed{\cos n\pi = (-1)^n}$

$\boxed{\cos(\text{even}.\pi) = 1}$ or $\boxed{\cos(2n\pi) = 1}$

$\boxed{\cos(\text{odd}.\pi) = -1}$ or $\boxed{\cos(2n\pi+1) = \cos(2n\pi-1) = -1}$

(IV) (i) $\int x^n \cdot dx = \frac{x^{n+1}}{n+1}$ | (iii) $\int 1 \cdot dx = x + C$

(ii) $\int \frac{1}{\sqrt{x}} \cdot dx = 2\sqrt{x}$ | (iv) $\int \frac{1}{x^2} \cdot dx = \frac{-1}{x}$

(v) $\int \frac{1}{x} \cdot dx = \log|x|$

(vi) $\int e^x \cdot dx = e^x$

(vii) $\int a^x \cdot dx = \frac{a^x}{\log a}$

most Imp

(viii) $\boxed{\int \cos x \cdot dx = \sin x}$

(ix) $\boxed{\int \sin x \cdot dx = -\cos x}$

(x) $\int \sec^2 x \cdot dx = \tan x$

(11) $\int \sec x \cdot \tan x \cdot dx = \sec x$

(12) $\int \operatorname{cosec}^2 x \cdot dx = -\cot x$

(13) $\int \operatorname{cosec} x \cdot \cot x \cdot dx = -\operatorname{cosec} x$

(14) $\int \frac{1}{\sqrt{1-x^2}} \cdot dx = \sin^{-1} x$

(15) $\int \frac{-1}{\sqrt{1-x^2}} \cdot dx = \cos^{-1} x$

(16) $\int \frac{1}{1+x^2} \cdot dx = \tan^{-1} x$

(17) $\int \frac{-1}{1+x^2} \cdot dx = \cot^{-1} x$

(18) $\int \frac{1}{x\sqrt{x^2-1}} \cdot dx = \sec^{-1} x$

(19) $\int \frac{-1}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x$

(20) $\int \tan^2 x \cdot dx = \int (\sec^2 x - 1) dx$

(21) $\int \cot^2 x \, dx = \int (\operatorname{cosec}^2 x - 1) dx$

(22) $\int \sin^2 x \cdot dx = \int \left(\frac{1-\cos 2x}{2}\right) dx$

(23) $\int \cos^2 x \cdot dx = \int \left(\frac{1+\cos 2x}{2}\right) dx$

(24) $\int \sin^3 x \cdot dx = \int \frac{3\sin x - \sin 3x}{4} dx$

(25) $\int \cos^3 x \cdot dx = \int \frac{3\cos x + \cos 3x}{4} dx$

(26) $\int \frac{f'(x)}{f(x)} \cdot dx = \log f(x) + C$

(27) $\int (u \cdot v) dx = u \int v \cdot dx - \int \left(\frac{du}{dx} \cdot \int v \cdot dx\right) dx$

ILATE for u, v

(28) $\int e^x \{f(x) + f'(x)\} dx = e^x \cdot f(x)$

(29) $\int \tan x \cdot dx = \log|\sec x|$

(30) $\int \cot x \cdot dx = \log|\sin x|$

(31) $\int \sec x \, dx = \log|\sec x + \tan x|$

(32) $\int \operatorname{cosec} x \cdot dx = \log|\operatorname{cosec} x - \cot x|$

(33)

Integration by parts

$\boxed{\int (u \cdot v) dx =}$

* Dirichlet's Conditions

(i) $f(x)$ can be posible/expressed in fourier series if $f(x)$ is a preodic function, single value & finite.

(ii) $f(x)$ has finite number of discontinuites in any one period.

(iii) $f(x)$ has finite number of maxima and minima in any one period.

Properties of Definite Integrals

① $\int_a^b f(x)\,dx = \int_a^b f(z)\cdot dz$

$2\sin A\cdot\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\cdot\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cdot\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\cdot\sin B = \cos(A-B) - \cos(A+B)$

② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

③ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

④ $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

⑤ $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

⑥ $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(2a-x) = f(x) \\ \text{if } f(2a-x) = -f(x) & \text{then} = 0 \end{cases}$

⑦ $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(-x) = f(x) \\ \text{if } f(-x) = -f(x) & \text{then} = 0 \end{cases}$

* $\boxed{\text{Periodic} \Rightarrow f(x) = \sin x, \cos x}$

* $\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx}$ ← fourier series formula

* $\int (u\cdot v)\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\cdot\int v\,dx\right)dx$

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* $\int (u\cdot v)\,dx = (uv_1 - u'v_2 + u''v_3 - u'''v_4 + u^{iv}v_5 -)$ - - - - - -

* $\int e^{ax} \cdot \sin bx \cdot dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]$

* $\int e^{ax} \cdot \cos bx \cdot dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right]$

* Fourier Coefficients :- formula

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx}$$

(i) $\boxed{a_0 = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\,dx}$

(ii) $\boxed{a_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\cos nx \cdot dx}$

(iii) $\boxed{b_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\sin nx \cdot dx}$

Types of limits

→ type ① ⇒ $(0, 2\pi)$ ✓
→ type ② ⇒ $(-\pi, \pi)$ ✓
→ type ③ ⇒ $(0, 2a)$ ✓
→ type 4 ⇒ $(-l, l)$ ✓
→ Half Range function ✓

*before puting limits in funn you should out of constant term from function than put your limit

Type - 1

Q.1 ⇒ find the fourier series of $f(x) = x^2$ in $(0, 2\pi)$

⇒ let $x^2 = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$.

$a_n = \frac{1}{\pi}\int_{0}^{2\pi} \underset{u}{x^2} \cdot \underset{v}{\cos nx} \cdot dx$

$a_n = \frac{1}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right) - 2x\left(\frac{-\cos nx}{n^2}\right) + 2\left(\frac{-\sin nx}{n^3}\right)\right]_0^{2\pi}$

$$a_n = \frac{1}{\pi}\left[2x\left(\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$a_n = \frac{1}{\pi}\left[4\pi\left(\frac{\cos 2n\pi}{n^2}\right) - 0\right]$$

$$a_n = \frac{1}{\cancel{\pi}}\left[\frac{4\cancel{\pi}\times 1}{n^2}\right] \Rightarrow \frac{4}{n^2}$$

$$\boxed{a_n = \frac{4}{n^2}}$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} x^2 \cdot dx \Rightarrow a_0 = \frac{1}{2\pi}\int_0^{2\pi} x^2 \cdot dx$$

$$a_0 = \frac{1}{2\pi}\left[\frac{x^3}{3}\right]_0^{2\pi} \Rightarrow a_0 = \frac{1}{2\pi}\left[\frac{8\pi^3}{3} - 0\right]$$

$$a_0 = \frac{8\pi^3}{6\pi} \qquad \boxed{a_0 = \frac{4}{3}\pi^2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \cdot \sin nx \cdot dx$$

$$b_n = \frac{1}{\pi}\left[x^2\left(-\frac{\cos nx}{n}\right) - 2x\left(-\frac{\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$b_n = \frac{1}{\pi}\left[x^2\left(-\frac{\cos nx}{n}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$b_n = \frac{1}{\pi}\left[(2\pi)^2\left(-\frac{\cos 2n\pi}{n}\right) + 2\left(\frac{\cos 2n\pi}{n^3}\right) - 2\left(\frac{\cos 0}{n^3}\right)\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{-4\pi^2}{n} + \cancel{\frac{2}{n^3}} - \cancel{\frac{2}{n^3}}\right] \Rightarrow b_n = \frac{-4\pi^2}{\pi n}$$

$$\boxed{b_n = \frac{-4\pi}{n}}$$

$\therefore$ fourier series is

$$x^2 = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$\boxed{x^2 = \frac{4}{3}\pi^2 + \sum_{n=1}^{\infty} \frac{4}{n^2} \cos nx + \sum_{n=1}^{\infty} \left(-\frac{4\pi}{n}\right) \sin nx}$$

(*) find fourier series expansion of $f(x)$

$$f(x) = \left(\frac{\pi - x}{2}\right)^2 \quad (0 \le x \ge 2\pi)$$

ans $\Rightarrow$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{\pi - x}{2}\right)^2 \cdot dx$$

$$a_0 = \frac{1}{8\pi} \int_0^{2\pi} (\pi - x)^2 \cdot dx \Rightarrow a_0 = \frac{1}{8\pi} \left[\frac{(\pi - x)^3}{-3}\right]_0^{2\pi}$$

$$\Rightarrow \frac{1}{8\pi} \left[\frac{(\pi - 2\pi)^3}{-3} - \left(\frac{\pi - 0}{-3}\right)^3\right]$$

$$\Rightarrow a_0 = \frac{1}{8\pi} \left[\frac{\pi^3}{3} + \frac{\pi^3}{3}\right]$$

$$a_0 = \frac{2\pi^3}{3} \cdot \frac{1}{8\pi} \qquad a_0 = \frac{\pi^3}{12\pi}$$

$$\boxed{a_0 = \frac{\pi^2}{12}}$$

Now, $a_n = \frac{1}{\pi} \int_0^{2\pi} \left(\frac{\pi - x}{2}\right)^2 \cdot \cos nx \cdot dx$

ans =)

$$\boxed{\left(\frac{\pi - x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx}$$

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$$a_n = \frac{1}{4\pi}\left[(\pi - x)^2 \cancel{\left(\frac{\sin nx}{n}\right)} - 2(\pi - x)(-1)\left(-\frac{\cos nx}{n^2}\right) + 2(-1)(-1)\cdot \cancel{\left(-\frac{\sin nx}{n^3}\right)}\right]_0^{2\pi}$$

$$a_n = \frac{1}{4\pi}\left[-2(\pi - x)(-1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$a_n = \frac{-\cancel{2}\times 1}{\cancel{4}\pi}\left[(\pi - 2\pi)\left(\frac{\cos 2n\pi}{n^2}\right) - (\pi - 0)\left(\frac{\cos 0}{n^2}\right)\right]$$

$$a_n = \frac{-1}{2\pi}\left[-\pi \cdot \frac{1}{n^2} - \left(\frac{\pi}{n^2}\right)\right]$$

$$a_n = \frac{-1}{2\pi}\left[\frac{-\pi}{n^2} - \frac{\pi}{n^2}\right]$$

$$a_n = \frac{\cancel{2\pi}}{n^2} \cdot \frac{1}{\cancel{2\pi}} \qquad \boxed{a_n = \frac{1}{n^2}}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \left(\frac{\pi - x}{2}\right)^2 \cdot \sin nx \cdot dx$$

$$b_n = \frac{1}{4\pi}\left[(\pi - x)^2\left(-\frac{\cos nx}{n}\right) - 2(\pi - x)(-1)\cancel{\left(-\frac{\sin nx}{n^2}\right)} + 2(-1)(-1)\left(+\frac{\cos nx}{n^3}\right)\right]$$

$$b_n = \frac{1}{4\pi}\left[(\pi - x)^2\left(-\frac{\cos nx}{n}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

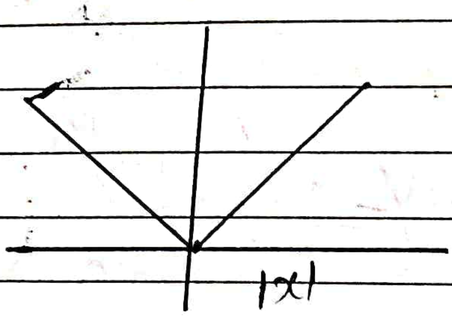
$$b_n = \frac{1}{4\pi}\left[(\pi - 2\pi)^2\left(-\frac{1}{n}\right) + 2\left(\frac{1}{n^3}\right)\right] - \left[(\pi - 0)^2\left(-\frac{1}{n}\right) + 2\left(\frac{1}{n^3}\right)\right]$$

$$b_n = \frac{1}{4\pi}\left[\frac{-\pi^2}{n} + \frac{2}{n^3}\right] - \left[\frac{-\pi^2}{n} + \frac{2}{n^3}\right]$$

$$b_n = \frac{1}{4\pi}\left[\cancel{\frac{-\pi^2}{n}} + \cancel{\frac{2}{n^3}} + \cancel{\frac{\pi^2}{n}} - \cancel{\frac{2}{n^3}}\right] \qquad \boxed{b_n = 0}$$

Note : $\boxed{\text{foureir Coefficients} \Rightarrow a_0, a_n, b_n}$

|x| graph →



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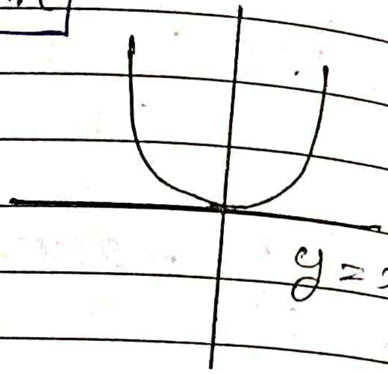

Note : (0) is an even function which is neither (+) nor ~~nither~~ (−)

Note : It is not necessry that all function can be categories into even and odd function.

* E + E = E
* odd + odd = odd → addition
* E × odd = odd
* Odd × Odd = E → multipilication
* E × E = E
* (O + E) or (E + O) → Nither even and nither odd → Excepatation

Note : Type II $(-\pi, \pi)$ in this if function is even then we have to find only (a_0) and (a_n) not need to find b_n because that is zero.

$$\boxed{\text{even} \rightarrow a_0 + a_n}$$

$$\boxed{\text{Odd} \rightarrow b_n}$$

Note : When you have to given limit $(-\pi \text{ to } \pi)$ then you can find odd & even method if limit is not $(-\pi, \pi)$ than not need to apply odd ~~ope~~ & even functions method.

* Type : II ⇒ $(-\pi, \pi)$.

* Expand the fourier series for periodic function.

$$f(x) = \begin{cases} -\pi & ----- -\pi < x < 0 \\ x & ---- 0 < x < \pi \end{cases}$$

ans ⇒ $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

$$-\pi \longleftrightarrow 0 \longleftrightarrow \pi$$

$$\cancel{a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot dx \Rightarrow a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} -\pi \cdot dx \Rightarrow a_0 = \frac{-\pi}{\pi}\left[x\right]_{-\pi}^{\pi}}$$

$$\cancel{a_0 = -1\left[\pi + \pi\right] \Rightarrow \boxed{a_0 = -2\pi}}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx \Rightarrow a_0 = \frac{1}{\pi}\int_{-\pi}^{0} -\pi \cdot dx + \int_{0}^{\pi} x \cdot dx$$

$$a_0 = \frac{-\pi}{\pi}\left[x\right]_{-\pi}^{0} + \left[\frac{x^2}{2}\right]_0^{\pi} \Rightarrow a_0 = \frac{-\pi}{\pi}\left[0 + \pi\right] + \left[\frac{\pi^2}{2} - 0\right]$$

$$a_0 = \frac{-\pi^2}{\pi} + \frac{\pi^2}{2} \Rightarrow a_0 = \frac{1}{\pi}\left[-\pi^2 + \frac{\pi^2}{2}\right]$$

$$a_0 = \frac{1}{\pi}\left[\frac{-2\pi^2 + \pi^2}{2}\right] \Rightarrow a_0 = \frac{1}{\pi} \times \frac{-\pi^2}{2}$$

$$\boxed{a_0 = \frac{-\pi}{2}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} -\pi \cdot \cos nx \cdot dx + \int_{0}^{\pi} x \cdot \cos nx \cdot dx$$

$$a_n = \frac{-\pi}{\pi}\left[\frac{\sin nx}{n}\right]_{-\pi}^{0} + \left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$a_n = 0 + \left[x(0) - \left(-\frac{\overset{(-1)^n}{\cos n\pi}}{n^2}\right)\right] - \left[0 - \left(-\frac{\cos 0}{n^2}\right)\right]$$

$$a_n \Rightarrow 0 + 0 + \frac{(-1)^n}{n^2} - \frac{1}{n^2} \Rightarrow \boxed{a_n = \frac{1}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right]}$$

$$\boxed{a_n = \frac{(-1)^n - 1}{n^2\pi}}$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi \cdot \sin nx \cdot dx + \int_{0}^{\pi} x \cdot \sin nx \cdot dx\right]$$

$$b_n = \frac{1}{\pi}\left[-\pi\left(-\frac{\cos nx}{n}\right)_{-\pi}^{0} + \left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}\right]$$

$$b_n = \frac{-\pi}{\pi}\left[\left(\frac{-1}{n}\right) - \left(-\frac{\cos n\pi}{n}\right)\right] + \left[\pi\left(-\frac{(-1)^n}{n}\right) - (0) - (0 - 0)\right]$$

$$b_n = \frac{-\cancel{\pi}}{\cancel{\pi}}\left[\frac{-1}{n} + \left(\frac{(-1)^n}{n}\right)\right] + \frac{\cancel{\pi}}{\cancel{\pi}}\left(-\frac{(-1)^n}{n}\right)$$

$$b_n = \frac{-\cancel{\pi}}{\cancel{\pi}}\left[\frac{(-1) + (-1)^n}{n}\right] - \frac{(-1)^n\cancel{\pi}}{n\cancel{\pi}}$$

$$b_n = \frac{1 - (-1)^n}{n} - \frac{(-1)^n}{n} \qquad \boxed{b_n = \frac{1 - 2(-1)^n}{n}}$$

$$\boxed{f(x) = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \cos nx \cdot \frac{(-1)^n - 1}{n^2\pi} + \sum_{n=1}^{\infty} \frac{1 - 2(-1)^n}{n} \sin nx}$$

(*) find fourier expasion of $f(x) = x^2$, $(-\pi \le x \le \pi)$

ans ⇒ $f(x) = x^2$, $f(-x) = (-x)^2$

$f(-x) = x^2$

$\boxed{f(-x) = f(x)}$

because $f(-x) = f(x)$ so that
The given fun^n is an even fun^n.

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx}$$

$$\boxed{a_0 = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\,dx} \quad a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,dx \quad \left[\because \int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx\right.$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x^2\,dx$$

$$a_0 = \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi} \Rightarrow a_0 = \frac{2}{\pi}\left[\frac{\pi^3}{3} - 0\right]$$

$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

$$a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\cdot\cos nx \Rightarrow a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx\,dx \quad \left|\; \because \int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx \text{ if } f(-x) = f(x)\right.$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx\,dx$$

$$a_n = \frac{2}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right) - 2x\left(-\frac{\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$a_n = \frac{2\times 2}{\pi}\left[\frac{\pi(-1)^n}{n^2}\right] \Rightarrow \boxed{a_n = \frac{4(-1)^n}{n^2}}$$

$$\boxed{x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2}\cos nx} \rightarrow \text{answer}$$

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① obtain the fourier series for.

$f(x) = x+\frac{\pi}{2}$; $-\pi < x < 0$

$= \frac{\pi}{2}-x$, $0 < x < \pi$

ans ⇒ if $f(-x) = (-x)+\frac{\pi}{2}$, $-\pi < -x < 0$

$= \frac{\pi}{2}-(-x)$, $0 < -x < \pi$

$f(-x) = \frac{\pi}{2}-x$, $-\pi < -x < 0$

$= \frac{\pi}{2}+x$, $0 < -x < \pi$

$f(-x) = \frac{\pi}{2}-x$, $\pi > x > 0$

$= \frac{\pi}{2}+x$, $0 > x \geq \pi$

$\boxed{f(-x) = f(x)}$ so that given f^n is even f^n

$\boxed{b_n = 0}$ $\boxed{a_n = ?}$ $\boxed{a_0 = ?}$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0}\left(x+\frac{\pi}{2}\right)dx + \int_{0}^{\pi}\left(\frac{\pi}{2}-x\right)dx\right]$$

So that there is a property that is

$$= \int_a^b f(x).dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} f(x).dx\right]$$

again There is a property that is $\int_{-a}^{a} f(x) = 2\int_0^a f(x)\,dx$

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$$a_0 = \frac{1}{\pi}\left[2\int_0^{\pi} f(x)\cdot dx\right]$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi}\left(\frac{\pi}{2} - x\right)dx$$

$$a_0 = \frac{2}{\pi}\left[\frac{\pi}{2}(x) - \frac{x^2}{2}\right]_0^{\pi} \Rightarrow \frac{2}{\pi}\left[\cancel{\frac{\pi^2}{2}} - \cancel{\frac{\pi^2}{2}}\right] - [0 - 0]$$

$$a_0 = \frac{2}{\pi}[0 - 0] \Rightarrow \boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\cdot \cos nx \cdot dx$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cdot \cos nx + \int_0^{\pi} f(x)\cos nx \cdot dx\right]$$

property $\Rightarrow \int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} f(x)\cdot \cos nx \cdot dx\right]$$ again property

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^{a} f(x)\cdot dx$$

$$\boxed{a_n = \frac{1}{\pi}\left[2\int_0^{\pi} f(x)\cdot \cos nx\, dx\right]}$$ $a_n = \frac{2}{\pi}\left[\int_0^{\pi}\left(\frac{\pi}{2} - x\right)\cos nx \cdot dx\right.$

$$a_n = \frac{2}{\pi}\left[\cancel{\left(\frac{\pi}{2} - x\right)\left(\frac{\sin nx}{n}\right)} - (-1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$a_n = \frac{2}{\pi}\left[\left(-\frac{(-1)^n}{n^2}\right) - \left(-\frac{1}{n^2}\right)\right] \Rightarrow a_n = \frac{2}{\pi}\left[-\frac{(-1)^n}{n^2} + \frac{1}{n^2}\right] \Rightarrow \boxed{a_n = \frac{2}{\pi}\left[\frac{1 - (-1)^n}{n^2}\right]}$$

$$\boxed{f(x) = 0 + \sum_{n=1}^{\infty} \frac{2(1-(-1)^n)}{n\pi^2} \cos nx}$$

ans ⇒

* find fourier series expansition for $f(x) = x^2$ for $-\pi \le x \le \pi$

ans ⇒ $f(x) = x^2, \quad -\pi \le x \le \pi$

if $f(-x) = (-x)^2 \Rightarrow f(-x) = x^2$

$f(-x) = f(x)$

so that given fun^n is even fun^n

$\boxed{b_n = 0}$ $\boxed{a_0, a_n = ?}$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x^2 \cdot dx\right] \Rightarrow a_0 = \frac{2}{\pi}\int_0^{\pi} x^2 \cdot dx$$

$$a_0 = \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi} \quad a_0 = \frac{2}{\pi}\left[\frac{\pi^3}{3} - 0\right]$$

$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

$$a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x) \cdot \cos nx \cdot dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cdot \cos nx \cdot dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x^2 \cdot \cos nx \cdot dx$$

$$a_n = \frac{2}{\pi}\left[\cancel{x^2\left(\frac{\sin nx}{n}\right)} - 2x\left(-\frac{\cos nx}{n^2}\right) + \cancel{2\left(-\frac{\sin nx}{n^3}\right)}\right]_0^{\pi}$$

$$a_n = \frac{2}{\pi}\left[-2x\left(\frac{-\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$a_n = \frac{-4}{\cancel{\pi}}\left[\cancel{\pi}\left(\frac{-(-1)^n}{n^2}\right) - 0\right]$$

$$\boxed{a_n = \frac{4(-1)^n}{n^2}}$$

$$\boxed{f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2}\cos nx}$$

(*) **find Fourier Series expansion of $f(x) = x - x^2$ $-\pi < x < \pi$** Danger $\boxed{f(x) = x + x^2}$

ans ⇒ $f(x) = x - x^2, \quad -\pi < x < \pi$

$f(-x) = (-x) - (-x)^2$
$f(-x) = -x - x^2$
$f(-x) = -(x + x^2)$

the given fn neither even nor odd

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} (x - x^2)\, dx$$

$$a_0 = \frac{1}{\pi}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-\pi}^{\pi} \Rightarrow \frac{1}{\pi}\left[\left(\frac{\pi^2}{2} - \frac{\pi^3}{3}\right) - \left(\frac{\pi^2}{2} + \frac{\pi^3}{3}\right)\right]$$

$$a_0 = \frac{1}{\pi}\left[\cancel{\frac{\pi^2}{2}} - \frac{\pi^3}{3} - \cancel{\frac{\pi^2}{2}} - \frac{\pi^3}{3}\right]$$

$$a_0 = \frac{-2\pi^{\cancel{3}\,2}}{3\cdot\cancel{\pi}}$$

$$\boxed{a_0 = \frac{-2\pi^2}{3}}$$

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$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x-x^2)\cdot \cos nx \cdot dx$$

$$a_n = \frac{1}{\pi}\left[\cancel{(x-x^2)\left(\frac{\sin nx}{n}\right)} - (1-2x)\left(-\frac{\cos nx}{n^2}\right) + \cancel{(-2)\left(-\frac{\sin nx}{n^3}\right)}\right]_{-\pi}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[(2x-1)\left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{\pi} \qquad a_n = \frac{1}{\pi}\left[(2\pi-1)\left(-\frac{(-1)^n}{n^2}\right) - (-2\pi-1)\left(\frac{(-1)^n}{n^2}\right)\right]$$

$$a_n = \frac{1}{\pi}\left[(2\pi-1)\left(-\frac{(-1)^n}{n^2}\right) - (-2\pi-1)\left(\frac{(-1)^n}{n^2}\right)\right]$$

$$a_n = \frac{1}{\pi}\Big[$$

$$a_n = \frac{1}{\pi}\left[\cancel{(x-x^2)\left(\frac{\sin nx}{n}\right)} - (1-2x)\left(-\frac{\cos nx}{n^2}\right) + \cancel{(-2)\left(\frac{\sin nx}{n^3}\right)}\right]_{-\pi}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[(1-2x)\left(\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[(1-2\pi)\left(\frac{\cos n\pi}{n^2}\right) - (1-2(-\pi))\left(\frac{\cos(-n\pi)}{n^2}\right)\right]$$

$$a_n = \frac{1}{\pi}\left[(1-2\pi)\frac{(-1)^n}{n^2} - (1+2\pi)\left(\frac{(-1)^n}{n^2}\right)\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{(-1)^n}{n^2}\left(\cancel{1}-2\pi-\cancel{1}-2\pi\right)\right]$$

$$a_n = \frac{1}{\cancel{\pi}}\left(\frac{(-1)^n}{n^2}\right)(-4\cancel{\pi}) \qquad \boxed{a_n = \frac{-4(-1)^n}{n^2}}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x - x^2)\cdot \sin nx \cdot dx$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x\cdot \sin nx - \int_{-\pi}^{\pi} x^2 \cdot \sin nx \, dx\right]$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x \cdot \sin nx - 0\right]$$

$$b_n = \frac{1}{\pi}\left[2\int_{0}^{\pi} x\cdot \sin nx \cdot dx\right]$$

$$b_n = \frac{2}{\pi}\left[(x)\left(-\frac{\cos nx}{n}\right) - \cancel{\left(-\frac{\sin nx}{n^2}\right)}\right]_0^{\pi}$$

$$b_n = \frac{2}{\pi}\left[\frac{-\pi(-1)^n}{n} - 0\right] \Rightarrow b_n = \frac{-2\cancel{\pi}(-1)^n}{n\cancel{\pi}}$$

$$\boxed{b_n = \frac{-2(-1)^n}{n}}$$

$$\boxed{x - x^2 = \frac{-\pi^2}{3} + \sum_{n=1}^{\infty} \frac{-4(-1)^n}{n^2}\cos nx + \frac{(-2)(-1)^n}{n}\sin nx}$$

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Type-3 ⇒ limits $(c \text{ to } c+2l) / (c \text{ to } c+2\pi)$

$$\boxed{l \to \text{interval} = \frac{(\text{final limit} - \text{Initial limit})}{2}}$$

$$\boxed{l = \frac{f - I}{2}}$$

∴ f.S →

$$\boxed{f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)}$$

$$\boxed{a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\cdot dx} \quad \boxed{a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cdot \cos\left(\frac{n\pi x}{l}\right)\cdot dx}$$

$$\boxed{b_n = \frac{1}{l}\int_c^{c+2l} f(x)\cdot \sin\left(\frac{n\pi x}{l}\right)\cdot dx}$$

Q. Find the fourier Expansition of x^2 in $(0, a)$

ans ⇒ $f(x) = x^2 \quad (0, a)$

$l = \frac{a-0}{2} \quad \boxed{l = \frac{a}{2}}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\cdot dx$$

$$a_0 = \frac{1}{\left(\frac{a}{2}\right)}\int_0^a x^2\, dx \Rightarrow a_0 = \frac{2}{a}\left[\frac{x^3}{3}\right]_0^a \Rightarrow a_0 = \frac{2}{a}\left[\frac{a^3}{3} - 0\right]$$

$$\boxed{a_0 = \frac{2a^2}{3}}$$

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$$a_n = \frac{2}{a}\int_0^a x^2 \cdot \cos\left(\frac{\cancel{2} n\pi x}{l}\right) \cdot dx$$

$$a_n = \frac{2}{a}\int_0^a x^2 \cdot \cos\left(\frac{2n\pi x}{a}\right) dx$$

$$a_n = \frac{2}{a}\left[x^2\cancel{\left(\frac{\sin\left(\frac{2n\pi x}{a}\right)}{\frac{2n\pi}{a}}\right)} - 2x\left(\frac{-\cos\left(\frac{2n\pi x}{a}\right)}{\left(\frac{2n\pi}{a}\right)^2}\right) + 2\left(\frac{-\sin\left(\frac{2n\pi x}{a}\right)}{\left(\frac{2n\pi}{a}\right)^3}\right)\right]_0^a$$

$$a_n = \frac{2}{a}\left[2x\left(\frac{\cos\left(\frac{2n\pi x}{a}\right)}{\left(\frac{2n\pi}{a}\right)^2}\right)\right]_0^a$$

$$a_n = \cancel{\frac{2\times 2}{a}}\left[2a\left(\frac{\cos\left(\frac{2n\pi a}{a}\right)}{\left(\frac{2n\pi}{a}\right)^2}\right)\right] - [0]$$

$$a_n = \frac{4a^2}{4n^2\pi^2} \qquad \boxed{a_n = \frac{\cancel{4}a^2}{\cancel{4}n^2\pi^2}} \qquad \boxed{a_n = \frac{a^2}{n^2\pi^2}}$$

$$\boxed{b_n = \frac{1}{l}\int_c^{c+2l} f(x) \cdot \sin\left(\frac{n\pi x}{l}\right) \cdot dx}$$

$$b_n = \frac{2}{a}\int_0^a x^2 \cdot \sin\left(\frac{n\pi x}{\frac{a}{2}}\right) \cdot dx$$

$$b_n = \frac{2}{a}\int_0^a x^2 \cdot \sin\left(\frac{2n\pi x}{a}\right) dx$$

$$b_n = \frac{2}{a}\left[x^2\left(\frac{-\cos\left(\frac{2n\pi x}{a}\right)}{\left(\frac{2n\pi}{a}\right)}\right) - (2x)\cancel{\left(\frac{-\sin\left(\frac{2n\pi x}{a}\right)}{\left(\frac{2n\pi}{a}\right)^2}\right)} + 2\left(\frac{+\cos\left(\frac{2n\pi x}{a}\right)}{\left(\frac{2n\pi}{a}\right)^3}\right)\right]_0^a$$

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$$b_n = \frac{-2}{a}\left[x^2\left(\frac{\cos\left(\frac{2n\pi x}{a}\right)}{\left(\frac{2n\pi}{a}\right)}\right) + 2x\cancel{\left(\frac{-\sin\left(\frac{2n\pi x}{a}\right)}{\left(\frac{2n\pi}{a}\right)^2}\right)} + 2\left(\frac{+\cos\left(\frac{2n\pi x}{a}\right)}{\left(\frac{2n\pi}{a}\right)^3}\right)\right]_0^a$$

$$b_n = \frac{-2}{a}\left[x^2\left(\frac{\cos\left(\frac{2n\pi x}{a}\right)}{\left(\frac{2n\pi}{a}\right)}\right) + 2\left(\frac{\cos\left(\frac{2n\pi x}{a}\right)}{\left(\frac{2n\pi}{a}\right)^3}\right)\right]_0^a$$

$$b_n = \frac{-2}{a}\left[\frac{a^2(1)}{\frac{2n\pi}{a}} + 2\left(\frac{1}{\left(\frac{2n\pi}{a}\right)^3}\right)\right] - \left(0 + 2\left(\frac{1}{\left(\frac{2n\pi}{a}\right)^3}\right)\right]$$

$$b_n = \frac{-2}{a}\left[\frac{a^3}{2n\pi} + \frac{2a^3}{8n^3\pi^3}\right] - \frac{2a^3}{8n^3\pi^3}$$

$$\boxed{\cancel{b_n = -\frac{2}{a}\left[\frac{a^3}{2n\pi}\left[\frac{1}{1} - \frac{a}{4n^2\pi^2}\right]\right] + 1\frac{a}{n\pi}} \quad \cancel{b_n = +\frac{2}{a}\left[\frac{a^3}{2n\pi}\left[\frac{4n^2\pi^2 - 2}{4n^2\pi^2}\right]\right] + \frac{a}{n\pi}}}$$

$$b_n = \frac{-2}{\cancel{a}}\left[\frac{a^{\cancel{3}}}{2n\pi} + \cancel{\frac{2a^3}{8n^3\pi^3}} - \cancel{\frac{2a^3}{8n^3\pi^3}}\right]$$

$$b_n = \frac{-\cancel{2}a^2}{\cancel{2}n\pi} \qquad \boxed{b_n = \frac{-a^2}{n\pi}}$$

$$\boxed{x^2 = \frac{a^2}{3} + \sum_{n=1}^{\infty}\frac{a^2}{n^2\pi^2}\cos\left(\frac{2n\pi x}{a}\right) + \sum_{n=1}^{\infty}\frac{-a^2}{n\pi}\sin\left(\frac{2n\pi x}{a}\right)}$$

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Q. Expansion of $f(x) = \begin{cases} \pi x, & 0<x<1 \\ 0, & 1<x<2 \end{cases}$ period 2 into a fourier series.

ans ⇒ $f(x) = \begin{cases} \pi x, & 0<x<1 \\ 0, & 1<x<2 \end{cases}$

$l = \frac{2-0}{2}$ $\boxed{l = 1}$

$a_0 = \frac{1}{l}\int_0^2 f(x)\cdot dx$ there is a property

$\int_a^b f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx$

$a_0 = \left[\int_0^1 \pi x\cdot dx + \int_1^2 0\cdot dx\right]$

$a_0 = \pi\left[\frac{x^2}{2}\right]_0^1 + 0 \Rightarrow \boxed{a_0 = \frac{\pi}{2}}$

$a_n = \int_0^2 f(x)\cdot \cos\left(\frac{n\pi x}{l}\right)\cdot dx$

$a_n = \int_0^1 \pi x\cdot dx\cdot\cos\left(\frac{n\pi x}{l}\right) + \int_1^0 0\cdot\cos\left(\frac{n\pi x}{l}\right) dx$

$a_n = \int_0^1 \pi x\cdot\cos\left(\frac{n\pi x}{l}\right)\cdot dx$

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$$a_n = \left[\cancel{\pi x\left(\sin\left(\frac{n\pi x}{n\pi}\right)\right)} - \pi\left(-\cos\left(\frac{n\pi x}{(n\pi)^2}\right)\right)\right]_0^1$$

$$a_n = \left[\pi\left(\frac{\cos n\pi x}{(n\pi)^2}\right)\right]_0^1$$

$$a_n = \left[\frac{\pi(-1)^n}{(n\pi)^2} - \frac{\pi}{(n\pi)^2}\right]$$

$$\boxed{a_n = \frac{\cancel{\pi}((-1)^n - 1)}{n^2\pi^{\cancel{2}}}} \quad \boxed{a_n = -\frac{(1-(-1)^n)}{n^2\pi}}$$

$$b_n = \int_0^2 f(x)\sin\left(\frac{n\pi x}{l}\right).dx$$

$$b_n = \int_0^1 \pi x.dx \, \sin\left(\frac{n\pi x}{l}\right) + \int_1^2 0.\sin\left(\frac{n\pi x}{l}\right).dx$$

$$b_n = \int_0^1 \pi x.\sin\left(\frac{n\pi x}{1}\right).dx + 0$$

$$b_n = \left[\pi x\left(-\cos\left(\frac{n\pi x}{n\pi}\right)\right) - \pi\left(-\sin\left(\frac{n\pi x}{(n\pi)^2}\right)\right)\right]_0^1$$

$$b_n = \left[-\pi\left(x\left(\cos\frac{n\pi x}{n\pi}\right) + \cancel{\pi\sin\left(\frac{n\pi x}{(n\pi)^2}\right)}\right)\right]_0^1$$

$$b_n = \left[-\pi\left(x\left(\frac{\cos n\pi x}{n\pi}\right)\right)\right]_0^1$$

$$b_n = -\frac{\cancel{\pi}(-1)^n}{n\cancel{\pi}} - (0) \quad \boxed{b_n = \frac{-(-1)^n}{n}}$$

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$$f(x) = \frac{\pi}{4} \sum_{n=1}^{\infty} \frac{-(1-(-1)^n)}{n^2\pi} \cos(n\pi x) + \sum_{n=1}^{\infty} \frac{-(-1)^n}{n} \sin(n\pi x)$$

* Obtain the f.s for the fun

$$f(x) = \begin{cases} \pi x & 0 \le x \le 1 \\ \pi(2-x) & 1 \le x \le 2 \end{cases}$$

$a_0 = \pi$ $\boxed{b_n = 0}$
$a_n = \frac{-4}{n^2\pi}, 0$

ans $l = \frac{2-0}{2}$ $\boxed{l = 1}$

$a_0 = \int_0^2 f(x) \cdot dx$ by property

$a_0 = \int_0^1 \pi x \cdot dx + \int_1^2 (2\pi - \pi x)\, dx$

$a_0 = \frac{\pi}{2} + \left[2\pi [x]_1^2 - \pi \left[\frac{x^2}{2}\right]_1^2 \right]$

$a_0 = \frac{\pi}{2} + \left[2\pi - \pi \left[\frac{4}{2} - \frac{1}{2} \right] \right]$ $a_0 = \frac{\pi}{2} + \left[2\pi - \pi \left[\frac{3}{2}\right] \right]$

$a_0 = \frac{\pi}{2} + \left[\frac{2\pi}{1} - \frac{3\pi}{2} \right]$

$a_0 = \frac{\pi}{2} + \left[\frac{4\pi - 3\pi}{2} \right] \Rightarrow a_0 = \frac{\pi}{2} + \left[\frac{\pi}{2}\right]$

$\boxed{a_0 = \pi}$

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$$a_n = \int_0^2 f(x)\cdot \cos\left(\frac{n\pi x}{1}\right)\cdot dx$$

$$a_n = \int_0^1 \pi x\cdot \cos(n\pi x)\cdot dx + \int_1^2 (2\pi - \pi x)\cdot \cos(n\pi x)\cdot dx$$

$$a_n = \frac{-(1-(-1)^n)\pi}{n^2\pi^2} + \int_1^2 (2\pi - \pi x)\cdot \cos(n\pi x)$$

$$= \left[(2\pi - \pi x)\left(\cancel{\frac{\sin(n\pi x)}{n\pi}}\right) - (-\pi)\left(\frac{-\cos(n\pi x)}{(n\pi)^2}\right)\right]_1^2$$

$$= \left[(2\pi - \pi x)\left(\cancel{\frac{\sin(n\pi x)}{n\pi}}\right) + \pi\left(\frac{-\cos(n\pi x)}{(n\pi)^2}\right)\right]_1^2$$

$$= \left(\frac{-\pi}{(n\pi)^2}\right) - \left(\pi\left(\frac{-(-1)^n}{(n\pi)^2}\right)\right)$$

$$\Rightarrow \frac{-(1-(-1)^n)\pi}{n^2\pi^2} + \frac{-\pi}{(n\pi)^2} + \frac{\pi(-1)^n}{(n\pi)^2}$$

$$\Rightarrow \frac{-\pi}{n^2\pi^2} + \frac{(-1)^n\pi}{n^2\pi^2} - \frac{\pi}{n^2\pi^2} + \frac{(-1)^n\pi}{n^2\pi^2}$$

$$\Rightarrow = \frac{2(-1)^n\pi}{n^2\pi^2} - \frac{2\pi}{n^2\pi^2} \Rightarrow \frac{2\pi}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$$= \frac{2((-1)^n - 1)}{n^2\pi} \Rightarrow \boxed{\frac{2((-1)^n - 1)}{n^2\pi}} \text{ if } n=1, n=2$$

$$\Rightarrow = \frac{-4}{n^2\pi}, 0$$

$$b_n = \int_0^2 f(x)\cdot \cos(n\pi x)\cdot dx$$

$$b_n = \int_0^1 \underset{u}{\pi x}\cdot \underset{v}{\sin(n\pi x)}\cdot dx + \int_1^2 \pi(2-x)\sin(n\pi x)\cdot dx$$

$$b_n = \pi\left[x\left(-\frac{\cos(n\pi x)}{n\pi}\right) - \left(\cancel{-\frac{\sin(n\pi x)}{(n\pi)^2}}\right)\right]_0^1 + \left[(2\pi - \pi x)\left(-\frac{\cos(n\pi x)}{(n\pi)}\right) - (-\pi)\left(\cancel{-\frac{\sin(n\pi x)}{(n\pi)^2}}\right)\right]_1^2$$

$$b_n = \pi\left[-\frac{(-1)^n}{n\pi}\right] + \left[(2\pi - 2\pi)\left(\frac{-1}{n\pi}\right) - (2\pi - \pi)\left(\frac{-(-1)^n}{n\pi}\right)\right]$$

$$b_n = \cancel{-\frac{(-1)^n\pi}{n\pi}} + \cancel{\frac{\pi(-1)^n}{n\pi}} \Rightarrow \boxed{b_n = 0}$$

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2((-1)^n - 1)\cos(n\pi x)}{n^2\pi}$$ [redacted]

(i) $f(x) = \int_0^{2\pi} e^{-x}\cdot dx$ (ii) $\int_{-l}^{l} e^{-x}\cdot dx$ (iii) $\int_{-\pi}^{\pi} e^{x}\cdot dx$

(iv) $f(x) = \int_0^1 e^{x}\cdot dx \rightarrow$ Half range series

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* obtain the half range series for e^x in interval $0<x<1$

$$f(x)=e^x \; , \; 0<x<1$$

ans ⇒

$$a_0=\frac{2}{l}\int_0^l f(x)\cdot dx \qquad l=f-I$$
$$l=1-0$$
$$l=1$$

$$a_0=\frac{2}{1}\int_0^1 e^x\cdot dx \Rightarrow a_0=2\left[e^x\right]_0^1$$

$$a_0=2[e^1-e^0]$$

$$\boxed{a_0=2(e-1)}$$

$$a_n=\frac{2}{l}\int_0^l f(x)\cdot\cos\left(\frac{n\pi x}{l}\right)dx$$

$$a_n=2\int_0^1 e^x\cdot\cos n\pi x\cdot dx \quad , a=1 , b=n\pi$$

$$a_n=2\left[\frac{e^x}{1^2+n^2\pi^2}\left(\cos n\pi x+n\pi\sin n\pi x\right)\right]_0^1$$

$$a_n=2\left[\left(\frac{e^x\cos n\pi x}{1+n^2\pi^2}+\cancel{\frac{e^x n\pi\sin n\pi x}{1+n^2\pi^2}}\right)\right]$$

$$a_n=2\left[\frac{e^x\cos n\pi x}{1+n^2\pi^2}\right]_0^1$$

$$a_n=2\left[\left(\frac{e^1(-1)^n}{1+n^2\pi^2}\right)-\left(\frac{e^0(1)}{1+n^2\pi^2}\right)\right]$$

$$a_n=2\left[\frac{(-1)^n e}{1+n^2\pi^2}-\frac{1}{1+n^2\pi^2}\right] \Rightarrow \boxed{a_n=\frac{2\left((-1)^n e-1\right)}{1+n^2\pi^2}}$$

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$$b_n = \frac{2}{l}\int_0^l f(x) \cdot \sin\left(\frac{n\pi x}{l}\right) \cdot dx$$ *

$$b_n = 2\int_0^1 e^x \cdot \sin\left(\frac{n\pi x}{l}\right) dx \quad , a = 1 , b = n\pi$$

$$b_n = 2\left[\frac{e^x}{1+n^2\pi^2}\left(\sin(n\pi x) - n\pi\cos(n\pi x)\right)\right]_0^1$$

$$b_n = 2\left[\cancel{\frac{e^x}{1+n^2\pi^2}\sin(n\pi x)} - \frac{e^x \cdot n\pi \cdot \cos(n\pi x)}{1+n^2\pi^2}\right]_0^1$$

$$b_n = -2\left(\frac{e^x}{1+n^2\pi^2} \cdot n\pi \cdot \cos(n\pi x)\right)_0^1$$

$$b_n = -2\left[\left(\frac{e^1}{1+n^2\pi^2} \cdot n\pi \cdot (-1)^n\right) - \left(\frac{e}{1+n^2\pi^2} \cdot n\pi \cdot 1\right)\right]$$

$$b_n = -2\left(\frac{(-1)^n n\pi e}{1+n^2\pi^2} - \frac{n\pi e}{1+n^2\pi^2}\right)$$

$$b_n = -2\left(\frac{(-1)^n n\pi e - n\pi e}{1+n^2\pi^2}\right)$$

$$\boxed{b_n = \frac{-2n\pi e\left((-1)^n - 1\right)}{1+n^2\pi^2}}$$

$$\boxed{b_n = \frac{2n\pi e}{1+n^2\pi^2}\left(1 - (-1)^n\right)}$$

* $f(x) = e^{-x} , \quad 0 < x < 2\pi$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} e^{-x} \cdot dx \Rightarrow a_0 = \frac{1}{\pi}\left[\frac{e^{-x}}{-1}\right]_0^{2\pi}$$

$$a_0 = -\frac{1}{\pi}\left[(e^{-2\pi}) - (e^{-0})\right] \Rightarrow \boxed{a_0 = \frac{-1}{\pi}\left[e^{-2\pi} - 1\right]}$$

$$\boxed{a_0 = \frac{1}{\pi}\left[1 - e^{-2\pi}\right]}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} e^{-x} \cdot \cos nx \cdot dx \quad , a = -1, \; b = n$$

$$a_n = \frac{1}{\pi}\left[\frac{e^{-x}}{(-1)^2 + n^2}\left(-\cos nx + n \sin nx\right)\right]_0^{2\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{-e^{-x}\cos nx}{1+n^2} + \cancel{\frac{e^{-x} n \sin nx}{1+n^2}}\right]_0^{2\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{-e^{-x}}{1+n^2}\cos nx\right]_0^{2\pi}$$

$$a_n = \frac{1}{\pi}\left[\left(\frac{-e^{-2\pi}}{1+n^2}\right) - \left(\frac{-e^{-0}(1)}{1+n^2}\right)\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{-e^{-2\pi}}{1+n^2} + \frac{1}{1+n^2}\right] \Rightarrow a_n = \frac{1}{\pi}\left[\frac{-e^{-2\pi}+1}{1+n^2}\right]$$

$$\boxed{a_n = \frac{1}{(1+n^2)\pi}\left(1 - e^{-2\pi}\right)}$$

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$$b_n = \frac{1}{\pi}\int_0^{2\pi} e^{-x} \cdot \sin nx \cdot dx \qquad a = -1, b = n$$

$$b_n = \frac{1}{\pi}\left[\frac{e^{-x}}{(-1)^2+n^2}\left(-\sin nx - n\cos nx\right)\right]_0^{2\pi}$$

$$b_n = \frac{1}{\pi}\left[\cancel{\frac{-e^{-x}\sin nx}{1+n^2}} - \frac{e^{-x} \cdot n\cos nx}{1+n^2}\right]_0^{2\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{-e^{-x} \cdot n \cdot \cos nx}{1+n^2}\right]_0^{2\pi}$$

$$b_n = \frac{1}{\pi}\left[\left(\frac{-e^{-2\pi} \cdot n \cdot 1}{1+n^2}\right) - \left(\frac{-e^{-0} \cdot n \cdot 1}{1+n^2}\right)\right]$$

$$b_n = \frac{1}{\pi}\left[\left(\frac{-ne^{-2\pi}}{1+n^2} + \frac{n}{1+n^2}\right)\right]$$

$$b_n = \left[\frac{1}{\pi}\left(\frac{-ne^{-2\pi}+n}{1+n^2}\right)\right]$$

$$\boxed{b_n = \frac{n}{\pi}\left(\frac{1-e^{-2\pi}}{1+n^2}\right)}$$

$$\boxed{b_n = \frac{n}{(1+n^2)\pi}\left(1-e^{-2\pi}\right)}$$

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* $\int_{-\pi}^{\pi} e^x \cdot dx$

$f(x) = e^x$

$f(-x) = e^{-x}$

$f(-x) \neq f(x)$

$f(-x) \neq -f(x)$

The given fun^n is neither even nor odd fun^n

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \cdot dx \Rightarrow a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \cdot dx$

$a_0 = \frac{1}{\pi}\left[e^x\right]_{-\pi}^{\pi} \Rightarrow \boxed{a_0 = \frac{\left[e^{\pi} - e^{-\pi}\right]}{\pi}}$ $\boxed{a_0 = \frac{2}{\pi}\sinh\pi}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \cdot \cos nx \cdot dx \Rightarrow a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \cdot \cos nx \, dx$

$a_n = \frac{1}{\pi}\left[\frac{e^x}{1+n^2}\left(\cos nx + n\sin nx\right)\right]_{-\pi}^{\pi}$

$a_n = \frac{1}{\pi}\left[\frac{e^x}{1+n^2}\cos nx + \cancel{\frac{e^x \cdot n \sin nx}{1+n^2}}\right]_{-\pi}^{\pi}$

$a_n = \frac{1}{\pi}\left[\frac{e^x}{1+n^2}\cos nx\right]_{-\pi}^{\pi}$

$a_n = \left(\frac{(-1)^n e^{\pi}}{1+n^2} - \frac{e^{-\pi}(-1)^n}{1+n^2}\right)\frac{1}{\pi}$

$\boxed{a_n = \frac{(-1)^n}{\pi}\frac{\left(e^{\pi} - e^{-\pi}\right)}{1+n^2}}$

$\therefore \boxed{\frac{e^x - e^{-x}}{2} = \sinh x}$

$\boxed{a_n = \frac{2(-1)^n \sinh\pi}{\pi(1+n^2)}}$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x . \sin nx . dx$$

$$b_n = \frac{1}{\pi}\left[\frac{e^x}{1+n^2}(\sin nx - n\cos nx)\right]_{-\pi}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[\cancel{\frac{e^x \sin nx}{1+n^2}} - \frac{e^x . n . \cos nx}{1+n^2}\right]_{-\pi}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[-\frac{e^x . n . \cos nx}{1+n^2}\right]_{-\pi}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[\left(\frac{-e^{\pi} n (-1)^n}{1+n^2}\right) + \left(\frac{e^{-\pi} n . (-1)^n}{1+n^2}\right)\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{-e^{\pi} n (-1)^n + n e^{-\pi} (-1)^n}{1+n^2}\right]$$

$$\boxed{b_n = \frac{n(-1)^n}{\pi} \frac{(-e^{\pi} + e^{-\pi})}{1+n^2}}$$

$$b_n = \frac{n(-1)^n}{(1+n^2)\pi} -2\left(\frac{e^{\pi} - e^{-\pi}}{2}\right)$$

$$\boxed{b_n = \frac{-2n(-1)^n \sinh\pi}{\pi(1+n^2)}}$$

$$\boxed{e^x = \frac{1}{\pi}\sinh\pi + \sum_{n=1}^{\infty} \frac{2(-1)^n \sinh\pi}{\pi(1+n^2)} + \sum_{n=1}^{\infty} \frac{-2(n)(-1)^n \sinh\pi}{\pi(1+n^2)}}$$

$$a_n = \frac{(-1)^n}{l}\left(\frac{-e^{-l}+e^{l}}{1+\left(\frac{n\pi}{l}\right)^2}\right)$$

$$a_n = \frac{(-1)^n\,(e^{l}-e^{-l})}{1+\frac{n^2\pi^2}{l^2}} \Rightarrow \frac{(-1)^n\,(e^{l}-e^{-l})}{\frac{l^2+n^2\pi^2}{l^2}}$$

$$\boxed{a_n = \frac{(-1)^n\, l^2\,(e^{l}-e^{-l})}{l^2+n^2\pi^2}}$$

$$b_n = \frac{1}{l}\int_{-l}^{l} e^{-x}\cdot \sin\left(\frac{n\pi x}{l}\right)\cdot dx$$

$$b_n = \frac{1}{l}\left[\frac{e^{-x}}{(-1)^2+\left(\frac{n\pi}{l}\right)^2}\left(-\sin\left(\frac{n\pi x}{l}\right)-\left(\frac{n\pi}{l}\right)\left(\cos\frac{n\pi x}{l}\right)\right)\right]_{-l}^{l}$$

$$b_n = \frac{1}{l}\left[\left(\cancel{\frac{-e^{-x}\sin\left(\frac{n\pi x}{l}\right)}{1+\left(\frac{n\pi}{l}\right)^2}} - \frac{e^{-x}\cdot\left(\frac{n\pi}{l}\right)}{1+\left(\frac{n\pi}{l}\right)^2}\cos\frac{n\pi x}{l}\right)\right]_{-l}^{l}$$

$$b_n = \frac{1}{l}\left[\frac{-e^{-x}\left(\frac{n\pi}{l}\right)}{1+\left(\frac{n\pi}{l}\right)^2}\cos\left(\frac{n\pi x}{l}\right)\right]_{-l}^{l}$$

$$b_n = \frac{1}{l}\left[\left(\frac{-e^{-l}}{1+\left(\frac{n\pi}{l}\right)^2}\left(\frac{n\pi}{l}\right)(-1)^n\right)-\left(\frac{-e^{l}}{1+\left(\frac{n\pi}{l}\right)^2}\cdot(-1)^n\right)\right]$$

$$b_n = \frac{1}{l}\left[\frac{-e^{-l}}{1+\left(\frac{n\pi}{l}\right)^2}\left(\frac{n\pi}{l}\right)(-1)^n + \frac{e^{l}(-1)^n}{1+\left(\frac{n\pi}{l}\right)^2}\right]$$

$$b_n = \frac{1}{l}\left[\frac{-e^{-l}\left(\frac{n\pi}{l}\right)(-1)^n + e^{l}(-1)^n}{1+\left(\frac{n\pi}{l}\right)^2}\right]$$

$$\boxed{b_n = \frac{1}{l}(-1)^n\left(\frac{\left(-e^{-l}\left(\frac{n\pi}{l}\right)+e^{l}\right)l^2}{l^2+n^2\pi^2}\right)}$$

Type-4 ⇒ limit (-l to l) या (-a, a)

Q Expand $F(x) = \begin{cases} 0, & -c < x < 0 \\ a, & 0 < x < c \end{cases}$ In the fourier series of period 2a.

⇒ $\boxed{f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum b_n \sin\left(\frac{n\pi x}{l}\right)}$

$a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x) \cdot dx$ $\quad l = \frac{c+c}{2}$ $\quad \boxed{l = \frac{2c}{2}}$ $\boxed{l = c}$

$a_0 = \frac{1}{c}\int_{-c}^{c} f(x)\, dx$ There is a property

$$\boxed{\int_a^b f(x) \cdot dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx}$$

$a_0 = \frac{1}{c}\left[\int_{-c}^{0} 0 \cdot dx + \int_0^c a \cdot dx\right] \Rightarrow a_0 = \frac{1}{c}\left[a(c-0)\right]$

$a_0 = \frac{ac}{c}$ $\quad \boxed{a_0 = a}$

$a_n = \frac{1}{c}\int_{-c}^{c} f(x)\, dx$

$a_n = \frac{1}{c}\int_{-c}^{0} 0 \cdot dx + \int_0^c a \cdot dx \cdot \cos\left(\frac{n\pi x}{l}\right)$ $\quad$ ~~$a_n = \frac{1}{c}\left[a(c-0)\right]$~~

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$$a_n = \frac{1}{c}\left[\int_0^c a \cdot \cos\left(\frac{n\pi x}{c}\right) \cdot dx\right]$$

$$a_n = \frac{1}{c}\left[a\left[\frac{\sin\left(\frac{n\pi x}{c}\right)}{\left(\frac{n\pi}{c}\right)}\right]_0^c\right]$$

$$a_n = \frac{1}{c}\left[a\left[\frac{0}{0}\right]\right] \qquad \boxed{a_n = 0}$$

$$b_n = \frac{1}{c}\int_{-c}^{c} f(x)\sin\left(\frac{n\pi x}{l}\right) \cdot dx$$

$$b_n = \frac{1}{c}\left[\int_{-c}^{0} 0 \cdot \sin\left(\frac{n\pi x}{l}\right) \cdot dx + \int_0^c a \cdot \sin\left(\frac{n\pi x}{l}\right) \cdot dx\right]$$

$$b_n = \frac{1}{c}\left[a\left[\frac{-\cos\left(\frac{n\pi x}{c}\right)}{\left(\frac{n\pi}{c}\right)}\right]_0^c\right]$$

$$b_n = -\frac{1}{c}\left[\left[\frac{a\cos(n\pi)}{\left(\frac{n\pi}{c}\right)}\right] - \left[\frac{+\cos(0)}{\left(\frac{n\pi}{c}\right)}\right]\right]$$

$$b_n = -\frac{1}{c}\left[\frac{a \cdot (-1)^n}{\frac{n\pi}{c}} - \frac{1}{\frac{n\pi}{c}}\right]$$

$$b_n = \frac{-1}{c}\left[\frac{a(-1)^n + 1}{\frac{n\pi}{c}}\right]$$

$$\boxed{b_n = -\frac{\left(-1 + a(-1)^n\right)}{n\pi}}$$

$$b_n = \frac{a\left(1 - (-1)^n\right)}{n\pi} \qquad \boxed{b_n = \frac{a\left(1 - (-1)^n\right)}{n\pi}}$$

fourier series is

$$\boxed{f(x) = \frac{a}{2} + \sum_{n=1}^{\infty} \frac{a(1-(-1)^n)}{n\pi} \sin\left(\frac{n\pi x}{c}\right)}$$

If n is even

$$f(x) = \frac{a}{2} + \sum_{n=1}^{\infty} \frac{a(1-(-1)^2)}{n\pi} \sin\left(\frac{n\pi x}{c}\right)$$

$$\boxed{f(x) = \frac{a}{2}}$$

If n is odd

$$\boxed{f(x) = \frac{a}{2} + \frac{2}{n\pi} \sin\left(\frac{n\pi x}{c}\right)}$$

* obtain the fourier expansion of x^2 from $x = (-l$ to $l)$

The given function is an even fun^n then $\boxed{b_n = 0}$

ans ⇒ $f(x) = x^2, (-l \text{ to } l)$

$l = \frac{l+l}{2}$ $l = \frac{2l}{2}$ $\boxed{l = l}$

$$a_0 = \frac{1}{l}\int_{-l}^{l} x^2 \, dx$$

$$a_0 = \frac{1}{l}\left[\frac{x^3}{3}\right]_{-l}^{l}$$ ~~$a_0 = \frac{1}{l}\frac{l^3}{3}$~~

$$a_0 = \frac{2}{l}\int_0^l x^2 \, dx \Rightarrow a_0 = \frac{2}{l}\left[\frac{l^3}{3} - 0\right]$$

$$\boxed{a_0 = \frac{2l^2}{3}}$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cdot \cos\left(\frac{n\pi x}{l}\right) dx$$

$$a_n = \frac{1}{l}\int_{-l}^{l} x^2 \cdot \cos\left(\frac{n\pi x}{l}\right)\cdot dx$$

$$a_n = \frac{2}{l}\int_{0}^{l} x^2 \cdot \cos\left(\frac{n\pi x}{l}\right)\cdot dx$$

$$a_n = \frac{2}{l}\left[\cancel{x^2\left(\frac{\sin\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}\right)} - 2x\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}\right) + \cancel{2\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}\right)}\right]_0^l$$

$$a_n = \frac{2}{l}\left[2x\left(\frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_0^l$$

$$a_n = \frac{2\cdot 2}{l}\cdot\frac{1}{\left(\frac{n\pi}{l}\right)^2}\left[l\cdot(-1)^n\right]$$

$$a_n = \frac{2\cdot 2 l^2}{\cancel{l}\cdot n^2\pi^2}\left[l(-1)^n\right] \Rightarrow a_n = \frac{2\cdot 2l\cdot l(-1)^n}{n^2\pi^2}$$

$$\boxed{a_n = \frac{2\cdot 2l^2(-1)^n}{n^2\pi^2}}$$

$$\boxed{a_n = \frac{4l^2(-1)^n}{n^2\pi^2}}$$

$$\boxed{f(x) = \frac{l^2}{3} + \sum_{n=1}^{\infty}\frac{4(-1)^n l^2}{n^2\pi^2}\cos\left(\frac{n\pi x}{l}\right)}$$

* find the fourier series for $f(x) = 1 - x^2$ in $(-1, 1)$

ans ⇒ $f(x) = 1 - x^2$ $\boxed{l = 1}$

$$a_0 = \int_{-1}^{1} (1 - x^2) \cdot dx$$

$f(-x) = 1 - (-x)^2$

$f(-x) = 1 - x^2$

$f(-x) = f(x)$

∴ the given fun^n is an even function

$$a_0 = 2 \int_0^1 (1 - x^2) \cdot dx$$

$$a_0 = 2\left[x - \frac{x^3}{3}\right]_0^1 \quad a_0 = 2\left[1 - \frac{1}{3}\right] \quad a_0 = 2 \cdot \frac{2}{3}$$

$$\boxed{a_0 = \frac{4}{3}}$$

$$a_n = \int_{-1}^{1} (1 - x^2) \cdot \cos\left(\frac{n\pi x}{l}\right)$$

$$a_n = 2 \int_0^1 (1 - x^2) \cdot \cos\left(\frac{n\pi x}{1}\right) \Rightarrow a_n = 2\left[\cancel{(1 - x^2)\left(\frac{\sin(n\pi x)}{n\pi}\right)} - (-2x)\left(-\frac{\cos(n\pi x)}{(n\pi)^2}\right) + \cancel{(-2)\left(-\frac{\sin(n\pi x)}{(n\pi)^3}\right)}\right]_0^1$$

$$a_n = -2\left[2x\left(\frac{\cos n\pi x}{(n\pi)^2}\right)\right]_0^1 \Rightarrow a_n = \frac{-4}{(n\pi)^2}\left[(-1)^n - 0\right]$$

$$a_n = \frac{-4}{(n\pi)^2}\left[(-1)^n - 0\right] \Rightarrow \boxed{a_n = \frac{4\left(0 - (-1)^n\right)}{n^2\pi^2}}$$

$$\boxed{a_n = \frac{-4(-1)^n}{n^2\pi^2}}$$

$$\boxed{f(x) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{-4(-1)^n}{n^2\pi^2} \cdot \cos(n\pi x)}$$

* Half Range series

(i) Half Range Cosine series
(ii) Half Range Sine series

Note → $\boxed{l = \text{final} - \text{initial}}$

$\boxed{l = f - I}$

* Half Range Cosine Series :

$$\boxed{a_0 = \frac{2}{l}\int_0^l f(x)\cdot dx}$$

$$\boxed{a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)\cdot dx}$$

* Half Range Sine Series :

$$\boxed{b_n = \frac{2}{l}\int_0^l f(x)\cdot \sin\left(\frac{n\pi x}{l}\right)\cdot dx}$$

Note : Half Range Fourier series is based on type-4 method $(-l, l)$

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Q find a Cosine series of period 2π to represent $\sin x$ in $(0 \le x \le \pi)$

ans ⇒ $f(x) = \sin x$ ------- $(0 \le x \le \pi)$

we have to find cosine series

$\boxed{a_0 = ?}$ $\boxed{a_n = ?}$ $\boxed{b_n = 0}$

because cosine series is given to me so we know that cos is an even fun^n so that $b_n = 0$

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{l}\right) + b_n \cos\left(\frac{n\pi x}{l}\right)}$$

$l = f - I$

$l = \pi - 0$

$\boxed{l = \pi}$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x).dx$$

$$a_0 = \frac{2}{l}\int_0^l f(x).dx \Rightarrow a_0 = \frac{2}{\pi}\int_0^{\pi} \sin x.dx$$

$$a_0 = \frac{2}{\pi}\left[-\cos x\right]_0^{\pi} \Rightarrow a_0 = \frac{-2}{\pi}[-1-1] \Rightarrow \boxed{a_0 = \frac{4}{\pi}}$$

$$\boxed{a_n = \frac{1}{l}\int_{-l}^{l} f(x).\cos\left(\frac{n\pi x}{l}\right).dx}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \sin x.\cos\left(\frac{n\pi x}{\pi}\right).dx$$

$$a_n = \frac{2}{l}\int_0^{\pi} \sin x \cdot \cos(nx)\cdot dx$$

$\left\{ \because 2\sin A \cos B = \sin(A+B) + \sin(A-B) \right.$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \frac{\sin(x+nx) + \sin(x-nx)}{2}dx$$

$$a_n = \frac{1}{\pi}\int_0^{\pi}\left[\sin(n+1)x + \sin(1-n)x\right]_0^{\pi}$$

$$a_n = \frac{1}{\pi}\left[-\frac{\cos(n+1)x}{n+1} - \frac{\cos(1-n)x}{1-n}\right]_0^{\pi}$$

$$a_n = -\frac{1}{\pi}\left[\frac{\cos(n+1)x}{n+1} + \frac{\cos(1-n)x}{1-n}\right]_0^{\pi}$$

$$a_n = -\frac{1}{\pi}\left[\left(\frac{\cos(n+1)\pi}{n+1} + \frac{\cos(1-n)\pi}{1-n}\right) - \left(\frac{1}{n+1} + \frac{1}{1-n}\right)\right]$$

$\boxed{\text{when } n = \text{even}}$

$$a_n = -\frac{1}{\pi}\left[\frac{-1}{n+1} - \frac{1}{1-n} - \frac{1}{n+1} - \frac{1}{1-n}\right]$$

$$a_n = -\frac{1}{\pi}\left[\frac{-2}{n+1} - \frac{2}{1-n}\right]$$

$$a_n = \frac{2}{\pi}\left[\frac{1-\cancel{n}+\cancel{n}+1}{(n+1)(1-n)}\right]$$

$$a_n = \frac{2}{\pi} \cdot \frac{2}{(n+1)(1-n)} \quad = a_n = \frac{4}{\pi(n+1)(1-n)}$$

$$a_n = \frac{4}{\pi(1+n)(1-n)}$$

$$a_n = \frac{4}{\pi(1-n^2)} \Rightarrow \boxed{a_n = \frac{4}{\pi(1-n^2)}} \quad a_n = \frac{-4}{\pi(n^2-1)}$$

$\boxed{\text{If } n = \text{odd then } a_n = 0}$

$$\boxed{\sin x = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(n^2-1)}\cos nx}$$

* Half Range SINE Series

Q ⇒ obtain half range sine series in $(0,\pi)$ for $x(\pi-x)$.

ans ⇒ $f(x) = x(\pi-x) \ldots\ldots (0,\pi)$

$\boxed{a_0=0}$, $\boxed{a_n=0}$ $\boxed{b_n=?}$

$\boxed{\begin{matrix} l=\pi-0 \\ l=\pi \end{matrix}}$

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)$$

$$b_n = \frac{2}{\pi}\int_0^\pi (\pi x - x^2)\sin\left(\frac{n\pi x}{\pi}\right)\cdot dx$$

$$b_n = \frac{2}{\pi}\int_0^\pi (\underset{u}{\pi x - x^2})\cdot \underset{v}{\sin(nx)}\cdot dx$$

$$b_n = \frac{2}{\pi}\left[(\pi x - x^2)\left(-\frac{\cos(nx)}{n}\right) - (\pi-2x)\cancel{\left(-\frac{\sin(nx)}{n^2}\right)} + (-2)\left(+\frac{\cos(nx)}{n^3}\right)\right]_0^\pi$$

$$b_n = \frac{2}{\pi}\left[(\pi^2-\pi^2)\left(\frac{-\cos n\pi}{n}\right) - 2\left(\frac{\cos n\pi}{n^3}\right)\right] - \left[(-2)\left(\frac{\cos 0}{n^3}\right)\right]$$

$$b_n = \frac{2}{\pi}\left[\frac{-2(-1)^n}{n^3} + \frac{2}{n^3}\right] \Rightarrow b_n = \frac{1}{\pi}\left[\frac{2}{n^3} - \frac{2(-1)^n}{n^3}\right]$$

$$b_n = \frac{2\times 2}{\pi}\left[\frac{1}{n^3} - \frac{(-1)^n}{n^3}\right] \Rightarrow \boxed{b_n = \frac{4(1-(-1)^n)}{\pi n^3}}$$

$$\boxed{f(x) = \sum_{n=1}^{\infty} \frac{4}{\pi}\left[\frac{(1-(-1)^n)}{n^3}\right]}$$

if n is even then $\boxed{b_n = 0}$

if n = odd then $\frac{8}{\pi n^3}$

(*) find half range cosine and sine series expansion for the function

$f(x) = x \cdot \sin x \quad , 0 < x < \pi$

ans ⇒ Cosine Series

$f(x) = x \cdot \sin x$ $\Big\}$ $l = \pi - 0$, $\boxed{l = \pi}$ $\quad a_0 = ?$ $\quad a_n = ?$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} x \cdot \sin x \cdot dx$$

$$a_0 = \frac{2}{\pi} \left[x(-\cos x) - (-\sin x) \right]_0^{\pi}$$

$$a_0 = \frac{2}{\pi} \left[\pi(-\cos\pi) \right]$$

$$a_0 = \frac{2}{\pi} \pi(-(-1))$$

$$\boxed{a_0 = 2}$$

$$a_0 = -\frac{2\pi}{\pi} \left[\pi(-1)^n + 0) - (0 + 0) \right]$$

$$a_0 = -2 \left[(-1)^n \right] \quad \boxed{a_0 = -2(-1)^n} \quad a_0$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} x \cdot \sin x \cdot \cos\left(\frac{n\pi x}{\pi}\right) \cdot dx$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} x \cdot \sin x \cdot \cos nx \cdot dx$$

$$a_n = \frac{2}{2\pi} \int_0^{\pi} x \left[2\sin x \cdot \cos nx \right] dx$$

$$a_n = \frac{1}{\pi} \int_0^{\pi} \left[x \left[\sin(x + nx) + \sin(x - nx) \right] \right]$$

$$a_n = \frac{1}{\pi} \int_0^{\pi} \left[x\sin(1+n)x + x\sin(1-n)x \right]$$

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$$a_n = \frac{1}{\pi}\left[x\left(-\frac{\cos(1+n)x}{1+n}\right) - \left(-\cancel{\frac{\sin(1+n)x}{(1+n)^2}}\right) + x\left(-\frac{\cos(1-n)x}{1-n}\right) - \left(-\cancel{\frac{\sin(1-n)x}{(1-n)^2}}\right)\right]_0^\pi$$

$$a_n = \frac{1}{\pi}\left[x\left(-\frac{\cos(1+n)x}{1+n}\right) + x\left(-\frac{\cos(1-n)x}{1-n}\right)\right]_0^\pi$$

$$a_n = \frac{1}{\pi}\left[\pi\left(-\frac{\cos(1+n)\pi}{1+n}\right) + \pi\left(-\frac{\cos(1-n)\pi}{1-n}\right)\right]$$

$$a_n = \frac{1}{\cancel{\pi}}\cancel{\pi}\left[-\frac{\cos(1+n)\pi}{1+n} + \frac{\cos(1-n)\pi}{1-n}\right]$$

$n = \text{even}$

$$a_n = \left[-\frac{\cos(3\pi)}{1+n} - \frac{\cos(1-2)\pi}{1-n}\right]$$

$$a_n = \left[\frac{1}{1+n} + \frac{1}{1-n}\right] = \frac{2}{1-n^2}$$

$$b_n = \frac{2}{\pi}\int_0^\pi x \cdot \sin x \cdot \sin nx \cdot dx$$

$$b_n = \frac{\cancel{2}}{\cancel{2}\pi}\int_0^\pi x\left[\cos(x-nx) + \cos(x+nx)\right] dx$$

$$b_n = \frac{1}{\pi}\int_0^\pi x\cos(1-n)x\,dx + \frac{1}{\pi}\int_0^\pi x\cos(1+n)x \cdot dx$$

$$b_n = \frac{1}{\pi}\left[x\left(\cancel{\frac{\sin(1-n)x}{(1-n)}}\right) - \left(-\frac{\cos(1-n)x}{(1-n)^2}\right)\right]_0^\pi + \frac{1}{\pi}\left[x\left(\cancel{\frac{\sin(1+n)x}{(1+n)}}\right) - \left(-\frac{\cos(1+n)x}{(1+n)^2}\right)\right]_0^\pi$$

$$b_n = \frac{1}{\pi}\left[\frac{\cos(1-n)\pi}{(1-n)^2}\right] + \frac{1}{\pi}\left[\frac{\cos(1+n)\pi}{(1+n)^2}\right]_0$$

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$$b_n = \frac{1}{\pi}\left[\frac{\cos(1-n)\pi}{(1-n)^2}\right] + \frac{1}{\pi}\left[\frac{\cos(1+n)\pi}{(1+n)^2}\right]$$

if $n = \text{even}$

$$b_n = \frac{1}{\pi}\left[\frac{\cos(-\pi)}{(1-n)^2}\right] + \frac{1}{\pi}\left[\frac{\cos(3\pi)}{(1+n)^2}\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{-1}{(1-n)^2}\right] + \frac{1}{\pi}\left[\frac{-1}{(1+n)^2}\right]$$

$$b_n = \frac{-1}{(1-n)^2\pi} + \frac{1}{(1+n)^2\pi} \qquad \boxed{b_n = 0}$$

$$b_n = \frac{-1}{\pi}\left[\frac{1}{(1-n)^2} + \frac{1}{(1+n)^2}\right]$$

$$b_n = \frac{-1}{\pi}\left[\frac{(1+n)^2 + (1-n)^2}{(1-n)^2(1+n)^2}\right]$$

$$b_n = -\frac{1}{\pi}\left[\frac{1+n^2+\cancel{2n}+1+n^2-\cancel{2n}}{(1-n)^2(1+n)^2}\right]$$

$$b_n = \frac{-1}{\pi}\left[\frac{2+2n^2}{(1-n)^2(1+n)^2}\right] \Rightarrow b_n = \frac{-2}{\pi}\frac{(1+n^2)}{(1-n)^2(1+n)^2}$$

$$b_n = -\frac{2}{\pi}\frac{(1+n^2)}{(1-n)^2(1+n)^2}$$

$$b_n = -\frac{1}{\pi}\left[\frac{1+n^2+2n-(1+n^2-2n)}{(1-n)^2(1+n)^2}\right]$$

$$b_n = -\frac{1}{\pi}\left[\frac{\cancel{1}+\cancel{n^2}+2n-\cancel{1}-\cancel{n^2}+2n}{(1-n)^2(1+n)^2}\right]$$

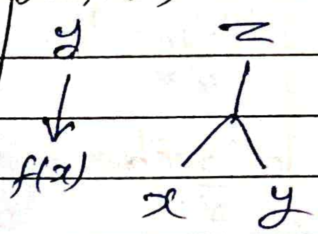
$$\boxed{b_n = \frac{4n}{(1-n)^2(1+n)^2}}$$

$$\boxed{(x^3+y^3+z^3-3xyz) = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)}$$

Partial Differentiation

$z = f(x,y)$ $\qquad y = f(x)$ $\qquad y=f(x)$

diff z partially w.r.to. x



$$\boxed{\frac{\partial z}{\partial x} = f'(x,y)}$$

diff z partially w.r.to. y

$$\boxed{\frac{\partial z}{\partial y} = f'(x,y)}$$

$$\boxed{(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca}$$

1. $\frac{d(x^n)}{dx} = nx^{n-1}$
2. $\frac{d(x)}{dx} = 1$
3. $\frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$
4. $\frac{d(\frac{1}{x})}{dx} = \frac{-1}{x^2}$
5. $\frac{d(e^x)}{dx} = e^x$
6. $\frac{d(a^x)}{dx} = a^x \cdot \log a$
7. $\frac{d(\sin x)}{dx} = \cos x$
8. $\frac{d(\cos x)}{dx} = -\sin x$
9. $\frac{d(\sin x)}{dx} = \cos x$
10. $\frac{d(\cos x)}{dx} = -\sin x$
11. $\frac{d(\tan x)}{dx} = \sec^2 x$
12. $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$
13. $\frac{d(\sec x)}{dx} = \sec x \cdot \tan x$
14. $\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cdot \cot x$
15. $\frac{d(c)}{dx} = 0$
16. $\frac{d(c \cdot u)}{dx} = c\frac{du}{dx}$
17. $\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$
18. $\frac{d(u \cdot v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$
19. $\frac{d(\frac{u}{v})}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
20. $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

(21) $\frac{d(\cos^{-1}x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$

(22) $\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$

(23) $\frac{d(\sec^{-1}x)}{dx} = \frac{1}{x\sqrt{x^2-1}}$

(24) $\frac{d(\operatorname{cosec}^{-1}x)}{dx} = \frac{-1}{x\sqrt{x^2-1}}$

(25) $\frac{d(\cot^{-1}x)}{dx} = \frac{-1}{1+x^2}$

(26) $\log_n m = \frac{\log m}{\log n}$

(27) $\log n^m = m \cdot \log n$

(28) $\log_a a = 1$ या $\log_e e = 1$

(29) $\log 0 = 1$

(33) $\log(m \times n) = \log m + \log n$

(30) $\log 1 = 0$

(31) $\log 2 = 0.3010$

(32) $\log_e = 2.303 \times \log_{10}$

(33) $\log\left(\frac{m}{n}\right) = \log m - \log n$

(1) If $u = e^x \cdot \sin x \cdot \sin y$, find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$

ans ⇒ $u = e^x \cdot \sin x \cdot \sin y$

Diff u partially w.r.to. x,

$$\frac{\partial u}{\partial x} = \sin y\,[e^x \cos x + \sin x \cdot e^x]$$

$$\boxed{\frac{\partial u}{\partial x} = e^x \cdot \sin y \cdot \cos x + \sin x \cdot \sin y\, e^x}$$

Similay, Diff u patially w.r.to

$$\frac{\partial u}{\partial y} = e^x \cdot \sin x\,[\cos y]$$

$$\boxed{\frac{\partial u}{\partial y} = e^x \cdot \sin x \cdot \cos y}$$

Properties of partial Diff :

1. If $\boxed{z = k}$, $\boxed{\dfrac{\partial z}{\partial x} = 0}$, $\boxed{\dfrac{\partial z}{\partial y} = 0}$

2. If $\boxed{z = f(y)}$, $\boxed{\dfrac{\partial z}{\partial x} = 0}$, [redacted]

3. If $\boxed{z = \sin(x+y)}$, $\dfrac{\partial z}{\partial x} = \cos(x+y)\,\dfrac{\partial(x+y)}{\partial x}$

$= \cos(x+y)\cdot 1$

4. $z = e^{x+y}$, $\boxed{\dfrac{\partial z}{\partial x} = e^{x+y}}$, $\boxed{\dfrac{\partial z}{\partial x} = e^{x+y}}$

5. $z = \log(x+y)$, $\boxed{\dfrac{\partial z}{\partial x} = \dfrac{1}{x+y}}$, $\boxed{\dfrac{\partial z}{\partial y} = \dfrac{1}{x+y}}$

partial diffrentiation

Q1. If [redacted] $z(x+y) = x-y$ find $\left(\dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}\right)^2$

ans ⇒ $z(x+y) = (x-y)$

$\boxed{z = \dfrac{x-y}{x+y}}$ diff z partially w.r.to. x

$\dfrac{\partial z}{\partial x} = \dfrac{(x+y)(1) - (x-y)(1)}{(x+y)^2}$, $\dfrac{\partial z}{\partial x} = \dfrac{\cancel{x}+y-\cancel{x}+y}{(x+y)^2}$

$\boxed{\dfrac{\partial z}{\partial x} = \dfrac{2y}{(x+y)^2}}$

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$$z = \frac{x-y}{x+y}$$

$$\frac{\partial z}{\partial y} = \frac{(x+y)(-1) - (x-y)(1)}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{-x-\cancel{y}-x+\cancel{y}}{(x+y)^2} \Rightarrow \boxed{\frac{\partial z}{\partial x} = \frac{-2x}{(x+y)^2}}$$

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2$$

$$\Rightarrow \left(\frac{2y}{(x+y)^2} + \frac{2x}{(x+y)^2}\right)^2 = \left(\frac{2x+2y}{(x+y)^2}\right)^2$$

$$\Rightarrow \left(\frac{2\cancel{(x+y)}}{(x+y)^{\cancel{2}}}\right)^2 \rightarrow$$

$$\Rightarrow \boxed{\frac{4}{(x+y)^2}}$$ answer.

~~If $z(x+y) = x-y$, find $\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}$~~

Q. If $z(x+y) = (x^2+y^2)$, prove that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

ans $\Rightarrow$ $$z = \frac{(x^2+y^2)}{(x+y)}$$

Diff z partially w.r.to. x

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$$\frac{\partial z}{\partial x} = \frac{(x+y)(2x) - (x^2+y^2)}{(x+y)^2}$$

$$\frac{\partial z}{\partial x} = \frac{2x^2+2xy-x^2-y^2}{(x+y)^2} = \frac{x^2+2xy-y^2}{(x+y)^2}$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{x^2+2xy-y^2}{(x+y)^2}}$$

$$\frac{\partial z}{\partial y} = \frac{(x+y)(2y) - (x^2+y^2)}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{2xy+2y^2-x^2-y^2}{(x+y)^2} \Rightarrow \boxed{\frac{2xy-x^2+y^2}{(x+y)^2}}$$

from

$\therefore$ L.H.S $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2$

$$\Rightarrow \left(\frac{(x^2+2xy-y^2)}{(x+y)^2} - \frac{(2xy-x^2+y^2)}{(x+y)^2}\right)^2$$

$$\Rightarrow \left(\frac{x^2+\cancel{2xy}-y^2-\cancel{2xy}+x^2-y^2}{(x+y)^2}\right)^2$$

$$\Rightarrow \left(\frac{2x^2-2y^2}{(x+y)^2}\right)^2 \Rightarrow \left(\frac{2(x^2-y^2)}{(x+y)^2}\right)^2$$

$$\Rightarrow \left(\frac{2\cancel{(x+y)}(x-y)}{\cancel{(x+y)^2}}\right)^2 \rightarrow \boxed{\frac{4(x-y)^2}{(x+y)^2}}$$

Similarly

R.H.S $\Rightarrow 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$

$$4\left(1 - \frac{(x^2+2xy-y^2)}{(x+y)^2} - \frac{(2xy-x^2+y^2)}{(x+y)^2}\right)$$

$$4\left(\frac{(x+y)^2 - x^2 - 2xy + y^2 - 2xy + x^2 - y^2}{(x+y)^2}\right)$$

$$\Rightarrow 4\left(\frac{x^2+y^2+2xy-x^2-2xy+y^2-2xy+x^2-y^2}{(x+y)^2}\right)$$

$$\Rightarrow \frac{4(-2xy+y^2+x^2)}{(x+y)^2}$$

$$\Rightarrow \frac{4(x^2+y^2-2xy)}{(x+y)^2} \Rightarrow \frac{4((x-y)^2)}{(x+y)^2}$$

$$\Rightarrow \frac{4(x-y)^2}{(x+y)^2}$$

$\therefore$ L.H.S = R.H.S

* properties of partial differentiation

$$\begin{tikzcd} & z \arrow[d] & \\ & u \arrow[dl] \arrow[dr] & \\ x & & y \end{tikzcd}$$

if $\boxed{z=f(u) \text{ and } u=\phi(x,y)}$

find $\boxed{\frac{\partial z}{\partial x} = \frac{dz}{du}\cdot\frac{\partial u}{\partial x}}$ $\boxed{\frac{\partial z}{\partial y} = \frac{dz}{du}\cdot\frac{\partial u}{\partial y}}$

(1) If $z = u^n$, then $\frac{\partial z}{\partial x} = \frac{dz}{du} \cdot \frac{\partial u}{\partial x}$

ans → $\boxed{\frac{\partial z}{\partial x} = n u^{n-1} \cdot \frac{\partial u}{\partial x}}$

$\boxed{\frac{\partial z}{\partial y} = n u^{n-1} \cdot \frac{\partial u}{\partial y}}$

* If $z = \sqrt{u}$, then $\frac{\partial z}{\partial x} = \frac{dz}{du} \cdot \frac{\partial u}{\partial x}$

$\boxed{\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{u}} \cdot \frac{\partial u}{\partial x}}$, $\boxed{\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{u}} \cdot \frac{\partial y}{\partial x}}$

* If $z = \sin u$, then $\frac{\partial z}{\partial x} = \frac{dz}{dx} \cdot \frac{\partial u}{\partial x}$

$\boxed{\frac{\partial z}{\partial x} = \cos u \cdot \frac{\partial u}{\partial x}}$, $\boxed{\frac{\partial z}{\partial y} = \cos u \cdot \frac{\partial u}{\partial y}}$

* If $u = \cos(\sqrt{x}+\sqrt{y})$ then prove that

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} + \frac{1}{2}(\sqrt{x}+\sqrt{y})\sin(\sqrt{x}+\sqrt{y}) = 0$$

ans → $\frac{\partial u}{\partial x} = -\sin(\sqrt{x}+\sqrt{y}) \cdot \frac{1}{2\sqrt{x}}$ —— (i)

$\frac{\partial u}{\partial y} = -\sin(\sqrt{x}+\sqrt{y}) \cdot \frac{1}{2\sqrt{y}}$ —— (ii)

$+y\frac{\partial u}{\partial y}$

∴ from L.H.S

$$x \cdot -\sin(\sqrt{x}+\sqrt{y}) \cdot \frac{1}{2\sqrt{x}} + y \cdot -\sin(\sqrt{x}+\sqrt{y}) \cdot \frac{1}{2\sqrt{y}} + \frac{1}{2}(\sqrt{x}+\sqrt{y}) \sin(\sqrt{x}+\sqrt{y}) = 0$$

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$$-\sin(\sqrt{x}+\sqrt{y})\cdot\frac{\sqrt{x}}{2}-\sin(\sqrt{x}+\sqrt{y})\cdot\frac{\sqrt{y}}{2}+\frac{1}{2}(\sqrt{x}+\sqrt{y})\cdot$$

$$\sin(\sqrt{x}+\sqrt{y})=0$$

$$\Rightarrow -\frac{1}{2}\sin(\sqrt{x}+\sqrt{y})\cancel{(\sqrt{x}+\sqrt{y})+\frac{1}{2}(\sqrt{x}+\sqrt{y})\sin(\sqrt{x}+\sqrt{y})}=0$$

$$=0$$

* If $u=\cos(\sqrt{x}+\sqrt{y}+\sqrt{z})$, prove that

$$x\cdot\frac{\partial u}{\partial x}+y\cdot\frac{\partial u}{\partial y}+z\cdot\frac{\partial u}{\partial z}+\frac{1}{2}(\sqrt{x}+\sqrt{y}+\sqrt{z})\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})=0$$

ans $\Rightarrow$ $\dfrac{\partial u}{\partial x}=-\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})\cdot\dfrac{1}{2\sqrt{x}}$ —— (i)

$\dfrac{\partial u}{\partial y}=-\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})\cdot\dfrac{1}{2\sqrt{y}}$ —— (ii)

$\dfrac{\partial u}{\partial z}=-\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})\cdot\dfrac{1}{2\sqrt{z}}$ —— (iii)

Put equations (i) and (ii) (iii) In L.H.S

$$x\cdot-\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})\cdot\frac{1}{2\sqrt{x}}+y\cdot-\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})\cdot\frac{1}{2\sqrt{y}}+z\cdot-\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})$$

$$\cdot\frac{1}{2\sqrt{z}}+\frac{1}{2}(\sqrt{x}+\sqrt{y}+\sqrt{z})\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})=0$$

$$-\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})\frac{\sqrt{x}}{2}-\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})\cdot\frac{\sqrt{y}}{2}-\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})\frac{\sqrt{z}}{2}$$

$$+\frac{1}{2}(\sqrt{x}+\sqrt{y}+\sqrt{z})\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})=0$$

$$\Rightarrow -\frac{1}{2}\cancel{\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})[\sqrt{x}+\sqrt{y}+\sqrt{z})+\frac{1}{2}(\sqrt{x}+\sqrt{y}+\sqrt{z})\cdot\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})}$$

$$=0$$

$$0=0$$

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③ If $u = (1-2xy+y^2)^{-1/2}$ prove that $x \cdot \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial x} = y^2 u^3$

ans ⇒ $\frac{\partial u}{\partial x} = -\frac{1}{2}(1-2xy+y^2)^{-3/2} \cdot (-2y)$

$$\boxed{\frac{\partial u}{\partial x} = (1-2xy+y^2)^{-3/2}(y)} \text{ — (i)}$$

$\frac{\partial u}{\partial y} = -\frac{1}{2}(1-2xy+y^2)^{-3/2}(-2x+2y)$

$\frac{\partial u}{\partial y} = -\frac{1}{2}(1-2xy+y^2)^{-3/2} \cdot -2(x-y)$

$$\boxed{\frac{\partial u}{\partial y} = (1-2xy+y^2)^{-3/2}(x-y)} \text{ — (ii)}$$

put eq (i) and (ii) in L.H.S

$x \cdot (1-2xy+y^2)^{-3/2} y - y(1-2xy+y^2)^{-3/2}(x-y) = y^2 u^3$

$x \cdot \left[(1-2xy+y^2)^{-1/2}\right]^3 y - y\left[(1-2xy+y^2)^{-1/2}\right]^3 (x-y) = y^2 u^3$

$\because \boxed{u = (1-2xy+y^2)^{-1/2}}$

$xy \cdot u^3 - y u^3(x-y) = y^2 u^3$

$xy \cdot u^3 - y(x-y)u^3 \Rightarrow u^3(xy - y(x-y))$

$\Rightarrow u^3(\cancel{xy} - \cancel{xy} + y^2)$

$\Rightarrow y^2 u^3$

R.H.S

* Differentiation of function of a function:

Ans ⇒ If $u = \log(\tan x + \tan y + \tan z)$, prove that

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$$

$u = \log(\tan x + \tan y + \tan z)$

diff u partially w.r.to. x

$$\frac{\partial u}{\partial x} = \frac{1}{\tan x + \tan y + \tan z}(\sec^2 x)$$

$$\boxed{\frac{\partial u}{\partial x} = \frac{\sec^2 x}{\tan x + \tan y + \tan z}}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\tan x + \tan y + \tan z}(\sec^2 y)$$

$$\boxed{\frac{\partial u}{\partial y} = \frac{\sec^2 y}{\tan x + \tan y + \tan z}}$$

$$\boxed{\frac{\partial u}{\partial z} = \frac{\sec^2 z}{\tan x + \tan y + \tan z}}$$

$$\sin 2x \frac{\partial u}{\partial x} = \sin 2x \cdot \frac{\sec^2 x}{\tan x + \tan y + \tan z}$$

$$\sin 2x \frac{\partial u}{\partial x} = \frac{1}{\tan x + \tan y + \tan z} \cdot \frac{2 \cdot \sin x \cdot \cancel{\cos x}}{\cos^{\cancel{2}} x}$$

$$\boxed{\sin 2x \cdot \frac{\partial u}{\partial x} = \frac{2 \tan x}{\tan x + \tan y + \tan z}} \quad (i)$$

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that

$$\boxed{\sin 2y \frac{\partial u}{\partial y} = \frac{2\tan y}{\tan x + \tan y + \tan z}} \quad \text{--- (ii)}$$

$$\boxed{\sin 2z \frac{\partial u}{\partial z} = \frac{2\tan z}{\tan x + \tan y + \tan z}} \quad \text{--- (iii)}$$

$\therefore$ from L.H.S

$$\Rightarrow \frac{2\tan x + 2\tan y + 2\tan z}{\tan x + \tan y + \tan z}$$

$$\Rightarrow \frac{2\cancel{(\tan x + \tan y + \tan z)}}{\cancel{(\tan x + \tan y + \tan z)}}$$

$$\Rightarrow 2$$

* If $u = \frac{e^{x+y+z}}{e^x + e^y + e^z}$, prove that $U_x + U_y + U_z = 2u$

$U_x = \frac{\partial u}{\partial x}$, $U_y = \frac{\partial u}{\partial y}$, $U_z = \frac{\partial u}{\partial z}$

$$\frac{\partial u}{\partial x} = \frac{(e^x + e^y + e^z)(e^{x+y+z}) - (e^{x+y+z})(e^x)}{(e^x+e^y+e^z)^2} \quad \text{--- (A)}$$

$$\frac{\partial u}{\partial x} = (e^{x+y+z})\left[\frac{\cancel{e^x} + e^y + e^z - \cancel{e^x}}{(e^x+e^y+e^z)^2}\right]$$

$$\boxed{\frac{\partial u}{\partial x} = (e^{x+y+z})\frac{[e^y + e^z]}{(e^x+e^y+e^z)^2}} \quad \text{--- (i)}$$

Similarly

$$\boxed{\frac{\partial u}{\partial y} = (e^{x+y+z})\frac{[e^x + e^z]}{(e^x+e^y+e^z)^2}} \quad \text{--- (ii)}$$

$$\boxed{\frac{\partial u}{\partial z} = \frac{e^{x+y+z}[e^x + e^y]}{(e^x+e^y+e^z)^2}} \quad \text{--- (iii)}$$

adding eq (i) + (ii) + (iii)

$$\Rightarrow \frac{e^{x+y+z}(e^{y}+e^{z})}{(e^x+e^y+e^z)^2} + \frac{e^{x+y+z}(e^{x}+e^{z})}{(e^x+e^y+e^z)^2} + \frac{e^{x+y+z}[e^{x}+e^{y}]}{(e^x+e^y+e^z)^2}$$

$$\Rightarrow \frac{e^{x+y+z}(e^{y}+e^{z})+e^{x+y+z}(e^{x}+e^{z})+e^{x+y+z}(e^{x}+e^{y})}{(e^x+e^y+e^z)^2}$$

$$\Rightarrow \frac{e^{x+y+z}[e^{y}+e^{z}+e^{x}+e^{z}+e^{x}+e^{y}]}{(e^x+e^y+e^z)^2}$$

$$\Rightarrow \frac{e^{x+y+z}[2e^{x}+2e^{y}+2e^{z}]}{(e^x+e^y+e^z)^2}$$

$$\Rightarrow e^{x+y+z}\cdot\frac{2\cancel{(e^x+e^y+e^z)}}{\cancel{(e^x+e^y+e^z)^2}}$$

$$\Rightarrow \frac{2\cdot e^{x+y+z}}{e^x+e^y+e^z} \quad \left\{ \because u = \frac{e^{x+y+z}}{e^x+e^y+e^z} \right.$$

$$\Rightarrow 2u$$

* If $u = x^2y + y^2z + z^2x$, Prove that
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x+y+z)^2$$

ans ⇒ Diff u partially w.r.to. x

$$\frac{\partial u}{\partial x} = 2xy + z^2 \text{ ——— (i)} \quad \text{adding (i)+(ii)+(iii)}$$

$$\frac{\partial u}{\partial y} = x^2 + 2yz \text{ ——— (ii)}$$

$$\frac{\partial u}{\partial z} = y^2 + 2xz \text{ ——— (iii)}$$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = z^2+2xy+x^2+2yz+y^2+2xz$$

$$= x^2+y^2+z^2+2xy+2yz+2xz$$

$\because$ $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$\Rightarrow$ $(x+y+z)^2$

* If $u=\log(x^2+y^2)$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y . \partial x}$

from L.H.S

ans $\Rightarrow$ $u=\log(x^2+y^2)$

$$\frac{\partial u}{\partial y} = \frac{1}{x^2+y^2}(2y) \quad , \quad \frac{\partial u}{\partial x} = \frac{1}{x^2+y^2}(2x)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = 2y\left(\frac{1}{x^2+y^2}\right) \Rightarrow 2y\left(\frac{-1}{(x^2+y^2)^2}\right)2x = \boxed{\frac{-4xy}{(x^2+y^2)^2}}$$

from R.H.S $\Rightarrow$

$$\frac{\partial u}{\partial x} = \frac{2x}{x^2+y^2} \quad , \quad \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = 2x\left(\frac{-1}{(x^2+y^2)^2}\right) \cdot 2y$$

$$\boxed{\frac{\partial^2 u}{\partial y . \partial x} = \frac{-4xy}{(x^2+y^2)^2}}$$

L.H.S = R.H.S

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(3) If $U = e^{x^2} + e^{y^2} + e^{z^2}$ prove that $\frac{\partial^3 U}{\partial x \cdot \partial y \cdot \partial z} = 8xyzU$.

Diff U partially w.r.to. x

$U = e^{x^2} + e^{y^2} + e^{z^2}$ $\frac{\partial U}{\partial x} = e$

$\frac{\partial U}{\partial z} = (e^{x^2} + e^{y^2} + e^{z^2})(2z)$

$\frac{\partial^2 U}{\partial y \cdot \partial z} = (e^{x^2} + e^{y^2} + e^{z^2})(2z)(2y)$

$\frac{\partial}{\partial y}\left(\frac{\partial U}{\partial z}\right) = (e^{x^2} + e^{y^2} + e^{z^2})(4yz)$

$\frac{\partial}{\partial x}\left(\frac{\partial^2 U}{\partial y \partial z}\right) = (e^{x^2} + e^{y^2} + e^{z^2})(4yz)(2x)$

$$\boxed{\frac{\partial^3 U}{\partial x \cdot \partial y \cdot \partial z} = 8xyzU}$$

* Euler's Theorem :-

Homogeneous function :

$$\boxed{U = x^n f\left(\frac{y}{x}\right)}$$

if U is homogenous fun^n of degree n then

$$\boxed{x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = nU}$$ n degree of x

it is also posiable :- $$\boxed{x \cdot \frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} + z \cdot \frac{\partial U}{\partial z} = nU}$$

* if $U = \frac{x^4+y^4}{x+y}$ then prove that

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nU}$$

by Euler theorem $\boxed{U = x^n f\left(\frac{y}{x}\right)}$

ans ⇒ $U = \frac{x^4+y^4}{x+y}$

$$U = \frac{\cancel{x^4}(1+y^4/x^4)}{\cancel{x}(1+y/x)}$$

$$U = x^3 f\left(\frac{1+(y/x)^4}{1+(y/x)^1}\right)$$

$\boxed{U = x^n f\left(\frac{y}{x}\right)}$ This is the homogeneous eqⁿ of degree $\boxed{n=3}$

∴ from Euler's formula

$$\boxed{x\cdot\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3U}$$ Hence proved.

function (x) defined 7·7·7·1

$$U = \log\left(\frac{x^4+y^4}{x+y}\right)$$

Note ÷ $\sin^{-1}x = \sin x$
$\cos x = \cos^{-1}x$
$\tan x = \tan^{-1}x$
like that → $\boxed{\log x = e^x}$

⇒ $U = \log\left(\frac{x^4+y^4}{x+y}\right)$

$$e^U = \frac{x^4+y^4}{x+y} \Rightarrow e^U = \frac{\cancel{x^4}(1+(y/x)^4)}{\cancel{x}(1+(y/x))}$$

$$\boxed{e^U = x^3 f(y/x)}$$

∴ from euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nU$$

$$x\cdot\frac{\partial}{\partial x}(e^U) + y\frac{\partial}{\partial y}(e^U) = 3e^x$$

$$x \cdot e^{u}\frac{\partial u}{\partial x}+y\cdot e^{u}\frac{\partial u}{\partial y}=3e^{u}$$

$$\cancel{e^{u}}\left[x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right]=3\cancel{e^{u}}$$

$$\boxed{x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3}$$ answer

* if $u=\tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then Prove that

$$\boxed{x\cdot\frac{\partial u}{\partial x}+y\cdot\frac{\partial u}{\partial y}=\sin 2u}$$ or $$\boxed{x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}= \sin 4u-\sin 2u}$$

$$u=\tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$$

$$\tan u=\frac{x^3+y^3}{x+y}$$

$$\tan u=\frac{x^3}{x}\left(\frac{1+(y/x)^3}{(1+y/x)^1}\right)$$

$$\boxed{\tan u=x^2 f\left(\frac{y}{x}\right)}$$ This is the homogeneous eq^n of degree $\boxed{n=2}$

$\therefore$ from Euler's formula $x\frac{\partial U}{\partial x}+y\frac{\partial U}{\partial y}=nU$

$$x\cdot\frac{\partial}{\partial x}(\tan u)+y\frac{\partial}{\partial y}(\tan u)=2\tan u$$

$$x\cdot\sec^2 u\cdot\frac{\partial u}{\partial x}+y\sec^2 u\frac{\partial u}{\partial y}=2\tan u$$

$$\sec^2 u\left[x\cdot\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right]=2\tan u$$

$$x\cdot\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{2\tan u}{\sec^2 u}$$

$$=\frac{2\cdot\sin u}{\cos u}\times\cos^2 u$$

$$=2\cdot\sin u\cdot\cos u$$

$$\boxed{x\cdot\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u}$$

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(x)

diff partially (i) with r. to. x both sides

$$x\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}+y\frac{\partial^2 u}{\partial x\cdot\partial y}=2\cos 2u\cdot\frac{\partial u}{\partial x}$$

Multiply with x both sides

$$x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial x\cdot\partial y}=2\cos 2u\frac{\partial u}{\partial x}-\frac{\partial u}{\partial x}$$

$$x^2\frac{\partial^2 u}{\partial x^2}+xy\frac{\partial^2 u}{\partial x\cdot\partial y}=(2\cos 2u-1)x\frac{\partial u}{\partial x} \quad \text{—— (A)}$$

again diff partially with r. to. y in eq (1) both sides

$$x\frac{\partial^2 u}{\partial x\cdot\partial y}+y\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}=2\cos 2u\cdot\frac{\partial u}{\partial y}$$

again

$$x\frac{\partial^2 u}{\partial x\cdot\partial y}+y\frac{\partial^2 u}{\partial y^2}=2\cos 2u\frac{\partial u}{\partial y}-\frac{\partial u}{\partial y}$$

again Multiply with y both sides

$$xy\frac{\partial^2 u}{\partial x\cdot\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(2\cos 2u-1)y\frac{\partial u}{\partial y} \quad \text{—— (B)}$$

add eq (A) + (B)

$$x^2\frac{\partial^2 u}{\partial x^2}+xy\frac{\partial^2 u}{\partial x\cdot\partial y}+xy\frac{\partial^2 u}{\partial x\cdot\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(2\cos 2u-1)x\cdot\frac{\partial u}{\partial x}+(2\cos 2u-1)y\frac{\partial u}{\partial y}$$

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\cdot\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(2\cos 2u-1)\left[x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right]$$

Teacher's Signature

from eq (1)

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\cdot\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(2\cos 2u-1)\sin 2u$$

$$= 2\cos 2u\cdot\sin 2u-\sin 2u$$

$$'' \quad '' \quad '' = \sin 4u-\sin 2u \quad (2\sin x\cdot\cos x=\sin 2x)$$

Hence proved

$$\boxed{x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\cdot\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\sin 4u-\sin 2u}$$

$$2\cos A\cdot\sin B=\sin(A+B)-\sin(A-B)$$

$$= \sin(3u+u)-\sin(3u-u)$$

$$= 2\cos 3u\cdot\sin u$$

$$\boxed{= 2\sin u\cdot\cos 3u}$$

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* if $u = \sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$ to such that

$$x\cdot\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} + 3\tan y = 0$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -3\tan y$$

$$u = \sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$$

$$u = \sin^{-1}\frac{1}{(x^8)^{\frac{1}{2}}}\left(\frac{x+2y+3z}{\sqrt{1+(y/x)^8+(z/x)^8}}\right)$$

$$u = \sin^{-1}\frac{x}{(x^8)^{\frac{1}{2}}}\left(\frac{1+2y/x+3z/x}{\sqrt{1+(y/x)^8+(z/x)^8}}\right)$$

$$\sin u = x\cdot x^{-4} f\left(\frac{y}{x}, \frac{z}{x}\right)$$

$$\sin u = x^{1-4} f\left(\frac{y}{x}, \frac{z}{x}\right)$$

$$\sin u = x^{-3} f\left(\frac{y}{x}, \frac{z}{x}\right)$$

$\boxed{\sin u = x^{-3} f\left(\frac{y}{x}, \frac{z}{x}\right)}$ This is homogeneous eq^n for degree $\boxed{n = -3}$

from ~~euler's theoreme~~ let $v = \sin u$

$$x\cdot\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + z\frac{\partial v}{\partial z} = nv$$

$$x\cdot\frac{\partial}{\partial x}(\sin u) + y\frac{\partial}{\partial y}(\sin u) + z\frac{\partial}{\partial z}(\sin u) = -3\sin u$$

$$x\cdot\cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial y} + z\cos u\frac{\partial u}{\partial z} = -3\sin u$$

$$\cos u\left[x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}\right]=-3\sin u$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=-\frac{3\sin u}{\cos u}$$

$$\boxed{x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=-3\tan u}$$

or $$\boxed{x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}+3\tan u=0}$$

If $u=\dfrac{x^2y}{x+y}$, then show that $\boxed{x\dfrac{\partial^2 u}{\partial x^2}+y\dfrac{\partial^2 u}{\partial y\cdot\partial x}=\dfrac{\partial u}{\partial x}}$

ans $\Rightarrow$ $$u=\frac{x^2y}{x+y}$$

$$u=\frac{\cancel{x^3}}{\cancel{x}}\left(\frac{y/x}{1+y/x}\right)\Rightarrow u=x^2f\left(\frac{y}{x}\right)$$

This is homogeneous eq. of degree $\boxed{n=2}$

$\therefore$ from Euler's Theorem $$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$$

$$x\cdot\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2\cdot u$$

partially diff w.r.to. x. both sides

$$x\cdot\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}+y\frac{\partial^2 u}{\partial x\cdot\partial y}=2\frac{\partial u}{\partial x}$$

$$x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial y\cdot\partial x}=2\frac{\partial u}{\partial x}-\frac{\partial u}{\partial x}\quad\boxed{x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial y\cdot\partial x}=\frac{\partial u}{\partial x}}$$

* If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

ans ⇒ $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$

∴ the degree of $x = 1$ and the degree of y is also $y = 1$ ~~so that total~~ its [illegible] means $\sin^{-1}$'s angle is 0 so that it is treated like constant, like that $\tan^{-1}\left(\frac{y}{x}\right)$

$\overset{0}{(\sin^{-1})}$ $\overset{0}{(\tan^{-1})}$

$u = f\left(\frac{x}{y}\right), \; f\left(\frac{1}{\frac{x}{y}}\right)$

$\boxed{u = x^{0} f\left(\frac{x}{y}\right)}$

so that u is the homogeneous fn of eq of $\boxed{n=0}$ degree

∴ from euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0 \cdot u$$

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0}$$ proved

* $u = \log\left(\frac{x^4 - y^4}{x - y}\right)$

prove that (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} =$

from 7.7.7.1 → $\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3}$

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$\Rightarrow$ $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$ — (1)

$\Rightarrow$ diff partially w.r.to. x. b.s

$x\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + y\frac{\partial^2 u}{\partial x \partial y} = 0$ —

multiply with x both sides

$x^2\frac{\partial^2 u}{\partial x^2} + xy\frac{\partial^2 u}{\partial x \partial y} = -x\frac{\partial u}{\partial x}$ — (A)

again diff eq (1) w.r.to. y b.s

$x\frac{\partial^2 u}{\partial y \cdot \partial x} + y\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} = 0$ —

again multiply with y both sides

$xy\frac{\partial^2 u}{\partial y \cdot \partial x} + y^2\frac{\partial^2 u}{\partial y^2} = -y\frac{\partial u}{\partial y}$ — (B)

add eq (A) + (B)

$x^2\frac{\partial^2 u}{\partial x^2} + xy\frac{\partial^2 u}{\partial x \cdot \partial y} + xy\frac{\partial^2 u}{\partial y \cdot \partial x} + y^2\frac{\partial^2 u}{\partial y^2} = -x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y}$

$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \cdot \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right]$

from eq (1)

$$\boxed{x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \cdot \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -3}$$ prove

if $u = (x^2+y^2)^{\frac{1}{3}}$, show that

$$x^2 \cdot \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{2}{9} u$$

ans $\Rightarrow$

$$u = (x^2+y^2)^{\frac{1}{3}}$$

$$u = (x^2)^{\frac{1}{3}} (1 + y^2/x^2)^{\frac{1}{3}}$$

$$u = x^{\frac{2}{3}} (1 + (y/x)^2)^{\frac{1}{3}}$$

$$u = x^{\frac{2}{3}} f\left(\frac{y}{x}\right)$$ so that u is a hommogenious fu^n of degree $\boxed{n = \frac{2}{3}}$

$\therefore$ from euler's theorme

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu$$

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{2}{3} u \quad \text{—— (i)}$$

again diff partially w.r. to. x both sides

$$x \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + y \frac{\partial^2 u}{\partial x \cdot \partial y} = \frac{2}{3} \frac{\partial u}{\partial x}$$

Multiplay with x both sides

$$x^2 \frac{\partial^2 u}{\partial x^2} + xy \frac{\partial^2 u}{\partial x \cdot \partial y} = \frac{2}{3} \frac{\partial u}{\partial x} x - x \frac{\partial u}{\partial x}$$

$$x^2 \frac{\partial^2 u}{\partial x^2} + xy \frac{\partial^2 u}{\partial x \cdot \partial y} = \frac{\partial u}{\partial x} \left[\frac{2}{3} x - x\right]$$

$$x^2 \frac{\partial^2 u}{\partial x^2} + xy \frac{\partial^2 u}{\partial x \cdot \partial y} = x \frac{\partial u}{\partial x} \left[\frac{2}{3} - \frac{1}{1}\right]$$

$$= x \frac{\partial u}{\partial x} \left(-\frac{1}{3}\right) \quad \text{—— (A)}$$

Again diff partially w.r.to. y b. sides in eq

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{2}{3}u$$

$$x\frac{\partial^2 u}{\partial y \cdot \partial x} + y\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} = \frac{2}{3}\frac{\partial u}{\partial y}$$

Multiply with y both sides

$$xy\frac{\partial^2 u}{\partial y \cdot \partial x} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{2}{3}y\frac{\partial u}{\partial y} - y\frac{\partial u}{\partial y}$$

$$xy\frac{\partial^2 u}{\partial y \cdot \partial x} + y^2\frac{\partial^2 u}{\partial y^2} = y\frac{\partial u}{\partial y}\left[\frac{2}{3} - 1\right] = \frac{-1}{3} \quad \text{(B)}$$

add eq (A) + (B)

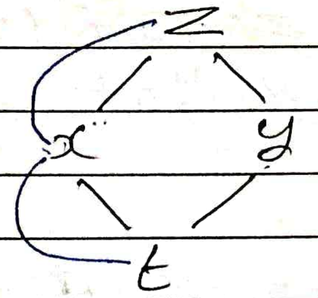
$$x^2\frac{\partial^2 u}{\partial x^2} + xy\frac{\partial^2 u}{\partial x \cdot \partial y} + xy\frac{\partial^2 u}{\partial y \cdot \partial x} + y^2\frac{\partial^2 u}{\partial y^2} = x\frac{\partial u}{\partial x}\left(\frac{-1}{3}\right) + y\frac{\partial u}{\partial y}\left(\frac{-1}{3}\right)$$

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \cdot \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{-1}{3}\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right]$$

from eq (1)

$$\boxed{x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \cdot \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{-2}{9}u}$$

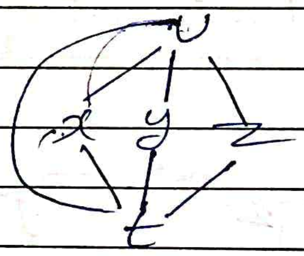
* Composite function :



z composite fun^n of t,
z fun^n of (x, y)
(x, y) fun^n of t

$$\boxed{\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}}$$

* $u = x^2 + y^2 + z^2$, $x = e^{2t}$, $y = e^{2t} \cos 3t$, $z = e^{2t} \sin 2t$
find $\boxed{\frac{du}{dt} = ?}$



$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt}$$

$$\frac{du}{dt} = 2x \cdot e^{2t} \cdot 2$$

$$\boxed{\frac{du}{dt} = 4x e^{2t}}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt}$$

$$\frac{dy}{dt} = e^{2t} \cos 3t$$

$$\boxed{\frac{dy}{dt} = 3e^{2t}(-\sin 3t) + 2\cos 3t \cdot e^{2t}}$$ How?

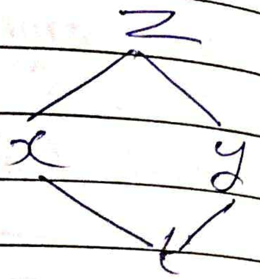
$$\frac{dz}{dt} = e^{2t} \sin 2t$$

$$\frac{dz}{dt} = 2e^{2t} \cos 2t + 2\sin 2t \, e^{2t}$$

$$\frac{du}{dt} = 4xe^{2t} + 2y(-3e^{2t} \sin 3t) + 2\cos 3t \cdot e^{2t} + 2z(2e^{2t}$$
$$\cos 2t + 2\sin 2t \cdot e^{2t}$$

* if $z = \sin^{-1}(x-y)$, $x = 3t$, $y = 4t^3$

prove that $\dfrac{dz}{dt} = \dfrac{3}{\sqrt{1-t^2}}$



ans $\Rightarrow$ $\boxed{\dfrac{dz}{dt} = \dfrac{\partial z}{\partial x} \cdot \dfrac{dx}{dt} + \dfrac{\partial z}{\partial y} \cdot \dfrac{dy}{dt}}$

$\boxed{\dfrac{\partial z}{\partial x} = \dfrac{1}{\sqrt{1-(x-y)^2}}}$, $\boxed{\dfrac{dx}{dt} = 3}$

$\boxed{\dfrac{\partial z}{\partial y} = \dfrac{-1}{\sqrt{1-(x-y)^2}}}$ $\boxed{\dfrac{dy}{dt} = 12t^2}$

$$\frac{dz}{dt} = \frac{1}{\sqrt{1-(x-y)^2}} \cdot 3 - \frac{1}{\sqrt{1-(x-y)^2}} \cdot 12t^2$$

$$\frac{dz}{dt} = \frac{3 - 12t^2}{\sqrt{1-(x-y)^2}}$$

$$\frac{dz}{dt} = \frac{3(1-4t^2)}{\sqrt{1-(3t-4t^3)^2}}$$

$$\frac{dz}{dt} = \frac{3(1-4t^2)}{\sqrt{1-(9t^2+16t^6-24t^4)}}$$

$$\frac{dz}{dt} = \frac{3(1-4t^2)}{\sqrt{1-9t^2-16t^6+24t^4}}$$

$$\frac{dz}{dt} = \frac{3(1-4t^2)}{\sqrt{1+16t^4+8t^4-8t^2-t^2-16t^6}}$$

$$\frac{dz}{dt} = \frac{3(1-4t^2)}{\sqrt{1+16t^4-8t^2+8t^4-t^2-16t^6}}$$

$\cancel{d\theta}$

$$\frac{dz}{dt} = \frac{3(1-4t^2)}{\sqrt{1+16t^4-8t^2-t^2(-8t^2+1+16t^4)}}$$

$$\frac{dz}{dt} = \frac{3(1-4t^2)}{\sqrt{1+16t^4-8t^2-t^2+1+16t^4-8t^2}}$$

$$\frac{dz}{dt} = \frac{3(1-4t^2)}{\sqrt{1+16t^4-8t^2(1-t^2)}}$$

$$\frac{dz}{dt} = \frac{3(1-4t^2)}{\sqrt{(1-4t^2)^2(1-t^2)}}$$

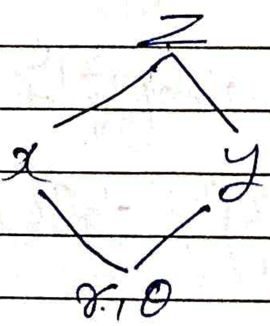
$$\frac{dz}{dt} = \frac{3\cancel{(1-4t^2)}}{\cancel{(1-4t^2)}\sqrt{1-t^2}}$$

$$\boxed{\frac{dz}{dt} = \frac{3}{\sqrt{1-t^2}}}$$

* if $z = f(x,y)$, $x = r\cos\theta$, $y = r\sin\theta$

prove that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$



$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial \theta}$$

$\frac{\partial x}{\partial r} = \cos\theta$, $\frac{\partial x}{\partial \theta} = -r\sin\theta$, $\frac{\partial y}{\partial \theta} = r\cos\theta$

$\frac{\partial y}{\partial r} = \sin\theta$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cos\theta + \sin\theta\frac{\partial z}{\partial y} \Rightarrow \left(\frac{\partial z}{\partial r}\right)^2 = \left(\cos\theta\frac{\partial z}{\partial x} + \sin\theta\frac{\partial z}{\partial y}\right)^2$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}(-r\sin\theta) + \frac{\partial z}{\partial y}(r\cos\theta)$$

$$\left(\frac{\partial z}{\partial \theta}\right)^2 = \left((-r\sin\theta)\frac{\partial z}{\partial x} + (r\cos\theta)\frac{\partial z}{\partial y}\right)^2$$

$$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

$$\Rightarrow \cos^2\theta\left(\frac{\partial z}{\partial x}\right)^2 + \sin^2\theta\left(\frac{\partial z}{\partial y}\right)^2 + 2\cdot\sin\theta\cos\theta\cdot\frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y} +$$
$$\frac{1}{r^2}\left(r^2\sin^2\theta\left(\frac{\partial z}{\partial x}\right)^2 + r^2\cos^2\theta\left(\frac{\partial z}{\partial y}\right)^2 - 2r\sin\theta\cdot r\cos\theta\,\frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y}\right)$$

$$\Rightarrow \cos^2\theta\left(\frac{\partial z}{\partial x}\right)^2 + \sin^2\theta\left(\frac{\partial z}{\partial y}\right)^2 + \cancel{2\sin\theta\cdot\cos\theta\frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y}} +$$
$$\cancel{\frac{1}{r^2}}\cdot\cancel{r^2}\left(\sin^2\theta\left(\frac{\partial z}{\partial x}\right)^2 + \cos^2\theta\left(\frac{\partial z}{\partial y}\right)^2 - \cancel{2\sin\theta\cdot\cos\theta\frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y}}\right)$$

$$\Rightarrow \cos^2\theta\left(\frac{\partial z}{\partial x}\right)^2 + \sin^2\theta\left(\frac{\partial z}{\partial y}\right)^2 + \sin^2\theta\left(\frac{\partial z}{\partial x}\right)^2 + \cos^2\theta\left(\frac{\partial z}{\partial y}\right)^2$$

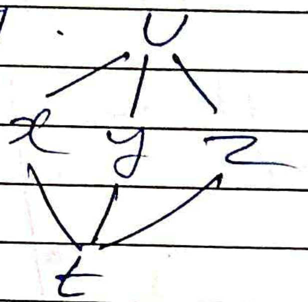
$$\Rightarrow \sin^2\theta\left(\frac{\partial z}{\partial y}\right)^2 + \cos^2\theta\left(\frac{\partial z}{\partial y}\right)^2 + \sin^2\theta\left(\frac{\partial z}{\partial x}\right)^2 + \cos^2\theta\left(\frac{\partial z}{\partial x}\right)^2$$

$$\Rightarrow \left(\frac{\partial z}{\partial y}\right)^2\left[\sin^2\theta + \cos^2\theta\right] + \left(\frac{\partial z}{\partial x}\right)^2\left[\sin^2\theta + \cos^2\theta\right]$$

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2.$$

L.H.S proned

(*) if $u = x^2 + y^2 + z^2$ and $x = e^{2t}$, $y = e^{2t}\cos 3t$, $z = e^{2t}\sin 3t$, find $\frac{du}{dt}$.



$$\frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial u}{\partial z}\cdot\frac{dz}{dt}$$

$$\boxed{\frac{\partial u}{\partial x} = 2x}, \boxed{\frac{\partial u}{\partial y} = 2y}, \boxed{\frac{\partial u}{\partial z} = 2z} \boxed{\frac{dx}{dt} = 2e^{2t}}$$

$$\boxed{\frac{dy}{dt} = -3e^{2t}\sin 3t + 2\cos 3t\, e^{2t}}$$

$$\boxed{\frac{dz}{dt} = 3e^{2t}\cos 3t + 2\sin 3t\, e^{2t}}$$

$$\frac{du}{dt} = \left(2x \cdot 2e^{2t}\right) + \left(2y\cdot(-3e^{2t}\sin 3t + 2\cos 3t \cdot e^{2t})\right) + \left(2z(3e^{2t}\cos 3t + 2\sin 3t\, e^{2t})\right)$$

$$\frac{du}{dt} = 4xe^{2t} - 6ye^{2t}\sin 3t + 4ye^{2t}\cos 3t + 6ze^{2t}\cos 3t + 4z\sin 3t\, e^{2t})$$

$$\frac{du}{dt} = 2e^{2t}(2x - 3y\sin 3t + 2y\cos 3t + 3z\cos 3t + 2z\sin 3t)$$

$$\frac{du}{dt} = 2e^{2t}\Big(2(e^{2t}) - 3(e^{2t}\cos 3t)\sin 3t + 2(e^{2t}\cos 3t)\cos 3t + 3e^{2t}\sin 3t\cos 3t + 2e^{2t}\sin 3t \cdot \sin 3t\Big)$$

$$= 2e^{2t}\Big(2e^{2t} - 3e^{2t}\cos 3t \cdot \sin 3t + 2e^{2t}\cos 3t\cos 3t + 3e^{2t}\sin 3t\cos 3t + 2e^{2t}\sin 3t \cdot \sin 3t\Big)$$

$$\Rightarrow 2e^{2t}\left[2 - \cancel{3\cos 3t\sin 3t} + 2\cos 3t\cos 3t + \cancel{3\sin 3t\cos 3t} + 2\sin 3t\sin 3t\right]$$

$$\Rightarrow 2e^{4t}\left[2 + 2\cos^2 3t + 2\sin^2 3t\right] \Rightarrow 2e^{4t}\left[2 + 2(\sin^2 3t + \cos^2 3t)\right] \Rightarrow \boxed{8e^{4t}}$$

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$*$ Jacobian ÷ property $\boxed{\frac{\partial(x,y)}{\partial(u,v)} \times \frac{\partial(u,v)}{\partial(x,y)} = 1}$

or $\boxed{JJ' = 1}$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}_{2\times 2}$$

$*$ if $x^m y^n = u$ and $x^r y^s = v$ then find

$$\boxed{\frac{\partial(x,y)}{\partial(u,v)} = ?}$$

we cannot find now the given expression because we don't get the value of x, y so that we find $\frac{\partial(u,v)}{\partial(x,y)}$ because we get the value of u,v and also it is a Jacobian property so after find of this value we can do inverse of this ans and we get answer.

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \qquad u = x^m y^n \qquad v = x^r y^s$$

$\boxed{\frac{\partial u}{\partial x} = y^n m x^{m-1}}$ $\boxed{\frac{\partial u}{\partial y} = n x^m y^{n-1}}$

$\boxed{\frac{\partial v}{\partial x} = r y^s x^{r-1}}$ $\boxed{\frac{\partial v}{\partial y} = s x^r y^{s-1}}$

$$\frac{\partial(U,V)}{\partial(x,y)} = \begin{vmatrix} m y^{n} x^{m-1} & n x^{m} y^{n-1} \\ r y^{s} x^{r-1} & s x^{r} y^{s-1} \end{vmatrix}$$

$\Rightarrow m y^{n} \cdot x^{m-1} \cdot s x^{r} \cdot y^{s-1} - r y^{s} x^{r-1} \cdot n x^{m} y^{n-1}$

$\Rightarrow x^{m+r-1} y^{n+s-1} \cdot m \cdot s - x^{m+r-1} \cdot y^{n+s-1} \cdot r \cdot n$

$\Rightarrow x^{m+r-1} y^{n+s-1} [ms - rn]$

$\Rightarrow (ms - rn)\, x^{m} \cdot x^{r} \cdot x^{-1}\, y^{n} \cdot y^{s} \cdot y^{-1}$

$\Rightarrow \dfrac{(ms-rn)\, x^{m} \cdot x^{r} \cdot y^{n} \cdot y^{s}}{xy} \Rightarrow \dfrac{(ms-rn)(x^{m} \cdot y^{n} \cdot x^{r} \cdot y^{s})}{xy}$

$U = x^{m} \cdot y^{n}, \; V = x^{r} y^{s}$

$\Rightarrow$

$\Rightarrow \dfrac{(ms-rn)UV}{xy} \Rightarrow \dfrac{\partial(U,V)}{\partial(x,y)} = \dfrac{(ms-rn)UV}{xy}$

$\therefore$ through Jacobian property

$$\frac{\partial(U,V)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(U,V)} = 1$$

$$\boxed{\frac{\partial(x,y)}{\partial(U,V)} = \frac{\partial(x,y)}{\partial(U,V)}}$$

$$\boxed{\frac{\partial(x,y)}{\partial(U,V)} = \frac{xy}{(ms-rn)UV}}$$

Note : Jacobian property (II) if (x,y) are fun^{n} of (u,v)

& (U,V) ——— (r,s) ~~fun of~~ r,s

then x,y are composite fun

$$\frac{\partial(x,y)}{\partial(r,s)} = \frac{\partial(x,y)}{\partial(U,V)} \times \frac{\partial(U,V)}{\partial(r,s)}$$

* if x, y are fun^n of θr then

$x = r\cos\theta$, $\frac{\partial(x,y)}{\partial(r,\theta)} = \frac{\partial(r,\theta)}{\partial(x,y)}$
$y = r\sin\theta$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$\frac{\partial x}{\partial r} = \cos\theta$, $\frac{\partial x}{\partial \theta} = -r\sin\theta$

$\frac{\partial y}{\partial r} = \sin\theta$ $\frac{\partial y}{\partial \theta} = r\cos\theta$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$= r\cos^2\theta + r\sin^2\theta$

$\boxed{\frac{\partial(x,y)}{\partial(r,\theta)} = r}$ so that by the propery of Jacobian $JJ^{-1} = 1$

$$\boxed{\frac{\partial(r,\theta)}{\partial(x,y)} = \frac{1}{r}}$$

* Three order Jacobian

$$\frac{\partial(x,y,z)}{\partial(r,\theta,z)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{vmatrix}$$

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* $x = r\cos\theta$
$y = r\sin\theta$
$z = z$
then find $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$

$$\frac{\partial(x,y,z)}{\partial(r,\theta,z)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{vmatrix}$$

$$\frac{\partial(x,y,z)}{\partial(r,\theta,z)} = \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 0 - 0 + 1(r\cos^2\theta + r\sin^2\theta)$$

$$\boxed{\frac{\partial(x,y,z)}{\partial(r,\theta,z)} = r}$$

* if $u = x + 3y^2 - z^3$
$v = 4x^2yz$
$w = 2z^2 - xy$
then evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0)$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} 1 & 6y & -3z^2 \\ 8xyz & 4x^2z & 4x^2y \\ -y & -x & 4z \end{vmatrix}$$

at $(1,-1,0)$
$x = 1,\ y = -1,\ 0 = z$

$$\begin{vmatrix} 1 & -6 & 0 \\ 0 & 0 & -4 \\ 1 & -1 & 0 \end{vmatrix} = 0 - 0 + 4(-1+6)$$

$$= 4 \times 5 = 20$$

$$\boxed{\frac{\partial(u,v,w)}{\partial(x,y,z)} = 20}$$

* taylor series

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \cdots$$

* Expand $\log \sin x$ in powers of $(x-2)$ by taylor's theorem.

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \cdots$$

Given $f(x) = \log \sin x$

and $a = x-2$

$a = x = 2$

$\boxed{a = 2}$

$f(x) = \log \sin x$	$f(a) = \log \sin 2$
$f'(x) = \cot x$	$f'(a) = \cot 2$
$f''(x) = -\operatorname{cosec}^2 x$	$f''(a) = -\operatorname{cosec}^2 2$
$f'''(x) = 2\operatorname{cosec}^2 x \cdot \cot x$	$f'''(a) = 2\operatorname{cosec}^2 2 \cdot \cot 2$

$$\log \sin x = \frac{\log \sin 2}{1!} + \frac{(x-2)}{1!}\cot 2 + \frac{(x-2)^2}{2!}(-\operatorname{cosec}^2 2) + \frac{(x-3)^3}{3!}(2\operatorname{cosec}^2 2 \cdot \cot 2)$$

* Expand $\tan^{-1} x$ in powers of $(x - \frac{\pi}{4})$ by taylor's Theorem.

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \cdots$$

$a = x - \frac{\pi}{4} \Rightarrow a = x = \frac{\pi}{4}$

$\boxed{a = \frac{\pi}{4}}$

$f(x) = \tan^{-1}x$

$f'(x) = \frac{1}{1+x^2}$

$f''(x) = -(1+x^2)^{-2} \cdot 2x$

$f''(x) = \frac{-2x}{(1+x^2)^2}$

$\cancel{f''(x) = 2 \cdot 2(1+x^2)}$

$\boxed{f(a) = \tan^{-1}\frac{\pi}{4} = 1}$

$f'(a) = \frac{1}{1+(\frac{\pi}{4})^2} = \frac{1}{1+\frac{\pi^2}{16}}$

$f'(a) = \frac{1}{\frac{16+\pi^2}{16}} \Rightarrow \frac{1}{16+\pi^2} \times 16$

$\boxed{f'(a) = \frac{16}{16+\pi^2}}$

$f''(a) = \frac{-2 \times \frac{\pi}{4} \times (16)^2}{(16+\pi^2)^2}$

$f''(a) = \frac{-\frac{\pi}{2} \times (16)^2}{(\pi^2+16)^2}$ $f''(a) = \frac{-\frac{\pi}{2} \times 256}{(\pi^2+16)^2}$

$\boxed{f''(a) = \frac{-128\pi}{(\pi^2+16)^2}}$

$\tan^{-1}x = 1 + \frac{(x-\pi/4)}{1!}\frac{16}{16+\pi^2} + \frac{(x-\pi/4)^2}{2!}\frac{(-128\pi)}{(\pi^2+16)^2}$

* Expand $2x^3+7x^2+x-1$ in powers of $(x-2)$ by Taylor's Theorem.

Ans ⇒ $f(x) = f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \cdots$

$f(x) = 2x^3+7x^2+x-1$, $a = x-2$, $a \Rightarrow x = 2$ | $\boxed{a = x = 2}$ $\boxed{a=2}$

$f'(x) = 6x^2+14x+1$

$f''(x) = 12x+14$

$f'''(x) = 12$

$f'(a) = 53$

$f''(a) = 38$, $f(a) = 16+28+2-1$

$f'''(a) = 12$ $= 45$ $f(a) = 45$

$$\boxed{2x^3+7x^2+x-1 = 45 + \frac{(x-2)}{1!}53 + \frac{(x-2)^2}{2!}38 + \frac{(x-2)^3}{3!}12}$$

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* Expand $\sin x$ in powers of $(x-\frac{\pi}{2})$ by Taylor's theorem.

ans ⇒ $f(x) = f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \text{---}$

$f(x) = \sin x$ $\quad a = x - \frac{\pi}{2}$ $\quad \boxed{a = \frac{\pi}{2}}$

$a = x = \frac{\pi}{2}$

$f'(x) = \cos x$ $\quad f(a) = 1$

$f''(x) = -\sin x$ $\quad f'(a) = 0$ $\quad f'''(a) = 0$

$f'''(x) = -\cos x$ $\quad f''(a) = -1$

$$\boxed{\sin x = 1 + \frac{(x-\pi/2)}{1!}0 + \frac{(x-\pi/2)^2}{2!}(-1) + \frac{(x-\pi/2)^3}{3!}0 + \text{---}}$$

* Taylor series expansion for function of two variable.

$$f(x,y) = f(a,b) + \frac{1}{1!}(xf_x + yf_y) + \frac{1}{2!}(x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy})$$

$$+ \frac{1}{3!}(x^3 f_{xxx} + 3x^2 y f_{xxy} + 3xy^2 f_{xyy} + y^3 f_{yyy})$$

Note :-

$f_x = \frac{\partial}{\partial x}$	$f_y = \frac{\partial}{\partial y}$	$f_{xy} = \frac{\partial}{\partial x \partial y}$
$f_{xx} = \frac{\partial^2}{\partial x^2}$	$f_{yy} = \frac{\partial}{\partial y^2}$	$f_{xxy} = \frac{\partial}{\partial x \partial x \partial y}$
$f_{xxx} = \frac{\partial^3}{\partial x^3}$	$f_{yyy} = \frac{\partial}{\partial y^3}$	$f_{xyy} = \frac{\partial}{\partial x \partial y \partial y}$

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* Expand $e^x \sin y$ in power of x & y as for as terms of third degree.

ans ⇒

$$f(x,y) = f(a,b) + \frac{1}{1!}(xf_x + yf_y) + \frac{1}{2!}(x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy}) +$$
$$\frac{1}{3!}(x^3 f_{xxx} + 3x^2 y f_{xxy} + 3xy^2 f_{xyy} + y^3 f_{yyy})$$

$$f(x,y) = e^x \sin y$$
$$f(0,0) = 0$$

$f_x = e^x \sin y$	$f_y = e^x \cos y$	$f_{xy} = e^x \cos y$
$f_{xx} = e^x \sin y$	$f_{yy} = -e^x \sin y$	$f_{xxy} = e^x \cos y$
$f_{xxx} = e^x \sin y$	$f_{yyy} = -e^x \cos y$	$f_{xyy} = -e^x \sin y$

at point $(0,0)$

$f_x = 0$	$f_y = 1$	$f_{xy} = 1$
$f_{xx} = 0$	$f_{yy} = 0$	$f_{xxy} = 1$
$f_{xxx} = 0$	$f_{yyy} = -1$	$f_{xyy} = 0$

$$f(x,y) = 0 + (x(0) + y(1)) + \frac{1}{2}(x^2(0) + 2xy(1) + y^2(0)) + \frac{1}{3\times 2}(x^3(0) + 3x^2y(1) +$$
$$3xy^2(0) + y^3(-1))$$

$$f(x,y) = 0 + (y) + \frac{1}{\cancel{2}}(\cancel{2}xy) + \frac{1}{6}(3x^2y - y^3)$$

$$f(x,y) = y + xy + \frac{3x^2y - y^3}{6}$$

$$\Rightarrow f(x,y) = y + xy + \frac{\cancel{3}x^2y}{\cancel{6}\,2} - \frac{y^3}{6}$$

$$\boxed{f(x,y) = y + xy + \frac{x^2y}{2} - \frac{y^3}{6}}$$

* find the first terms of the expansion of the function $f(x,y) = e^x \log(1+y)$ in taylor series about $(0,0)$.

ans ⇒

$$f(x,y) = f(a,b) + \frac{1}{1!}(x f_x + y f_y) + \frac{1}{2!}(x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy})$$

$$\frac{1}{3!}(x^3 f_{xxx} + 3x^2 y f_{xxy} + 3xy^2 f_{xyy} + y^3 f_{yyy})$$

$$f(x,y) = e^x \log(1+y)$$
$$f(0,0) = 0$$

$f_x = e^x \log(1+y)$ | $f_y = e^x/1+y$ | $f_{xy} = e^x/1+y$

$f_{xx} = e^x \log(1+y)$ | $f_{yy} = -e^x/(1+y)^2$ | $f_{xxy} = e^x/1+y$

$f_{xxx} = e^x \log(1+y)$ | $f_{yyy} = \frac{2e^x}{(1+y)^3}$ | $f_{xyy} = -e^x/(1+y)^2$

at point $(0,0)$

$f_x = 0$ | $f_y = 1$ | $f_{xy} = 1$

$f_{xx} = 0$ | $f_{yy} = -1$ | $f_{xxy} = 1$

$f_{xxx} = 0$ | $f_{yyy} = 2$ | $f_{xyy} = -1$

$$f(x,y) = 0 + (x(0) + y(1)) + \frac{1}{2}(x^2(0) + 2xy(1) + y^2(-1)) +$$

$$\frac{1}{3\times 2}(x^3(0) + 3x^2y(1) + 3xy^2(-1) + y^3(2))$$

$$f(x,y) = y + \frac{1}{\cancel{2}} \times \cancel{2}xy - \frac{y^2}{2} + \frac{1}{\cancel{6}2} \times \cancel{3}x^2y - \frac{\cancel{3}xy^2}{\cancel{6}2} + \frac{\cancel{2}y^3}{\cancel{6}3}$$

$$f(x,y) = y + xy - \frac{y^2}{2} + \frac{1}{2}x^2y - \frac{xy^2}{2} + \frac{y^3}{3}.$$

$$\boxed{e^x \log(1+y) = y + xy - \frac{y^2}{2} + \frac{x^2y}{2} - \frac{xy^2}{2} + \frac{y^3}{3}}$$

→ for one variable

(1) $f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \frac{(x-a)^4}{4!} f''''(a) + \dots$

(2) for two variable

$(x f_x + y f_y)$ या $\left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}\right)$ — (i)

(2) $(a+b)^2 \rightarrow$ apply in eqn (i) $(a+b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow (\underbrace{x f_x}_{a} + \underbrace{y f_y}_{b})^2 = x^2 f_{xx} + y^2 f_{yy} + 2xy f_{xy}$

again apply $(a+b)^3 = a^3 + b^3 + 3a^2 b + 3ab^2$

$(\underbrace{x f_x}_{a} + \underbrace{y f_y}_{b})^3 = x^3 f_{xxx} + y^3 f_{yyy} + 3x^2 y f_{xxy} + 3xy^2 f_{xyy}$

→ for two variable

(2) $f(x,y) = f(a,b) + (x f_x + y f_y) + \frac{1}{2!}(x^2 f_{xx} + y^2 f_{yy} + 2xy f_{xy}) + \frac{1}{3!}(x^3 f_{xxx} + y^3 f_{yyy} + 3x^2 y f_{xxy} + 3xy^2 f_{xyy})$

* for $f(x+h, y+k)$

apply $(a+b)^2$

$(h f_x + k f_y)^2 = h^2 f_{xx} + k^2 f_{yy} + 2hk f_{xy}$

$(h f_x + k f_y)^3 = h^3 f_{xxx} + k^3 f_{yyy} + 3h^2 k f_{xxy} + 3hk^2 f_{xyy}$

(3) $f(x+h, y+k) = f(x,y) + (h f_x + k f_y) + \frac{1}{2!}(h^2 f_{xx} + k^2 f_{yy} + 2hk f_{xy}) + \frac{1}{3!}(h^3 f_{xxx} + k^3 f_{yyy} + 3h^2 k f_{xxy} + 3hk^2 f_{xyy})$

* Expand : $f(x,y) = x^2y + 3y - 2$ in powers of $(x-1)$ & $(y+2)$ by Taylor's Theorem.

ans ⇒

$$f(x,y) = f(x,y) + (x f_x + y f_y) + \frac{1}{2!}(x^2 f_{xx} + y^2 f_{yy} + 2xy f_{xy}) + \frac{1}{3!}(x^3 f_{xxx} + y^3 f_{yyy} + 3x^2y f_{xxy} + 3xy^2 f_{xyy})$$

$f(x,y) = xy + 3y - 2$, $x - 1 = 0$, $y + 2 = 0$
$\boxed{x = 1}$, $\boxed{y = -2}$

$f(x,y) = x^2y + 3y - 2$
$f(1,-2) = (1)(-2) + 3(-2) - 2$
$f(1,-2) = -2 - 6 - 2$
$f(1,-2) = -10$

$f_x = 2yx$	$f_y = x^2 + 3$	$f_{xy} = 2x$
$f_{xx} = 2y$	$f_{yy} = 0$	$f_{xxy} = 2$
$f_{xxx} = 0$	$f_{yyy} = 0$	$f_{xyy} = 0$

at point (x,y) $(1,-2)$

$f_x = -4$	$f_y = 4$	$f_{xy} = 2$
$f_{xx} = -4$	$f_{yy} = 0$	$f_{xxy} = 2$
$f_{xxx} = 0$	$f_{yyy} = 0$	$f_{xyy} = 0$

$x^2y + 3y - 2 = -10 + (x(2xy) + y(x^2+3)) + \frac{1}{2}(x^2(2y) + y^2(0) + 2xy(2x))$
$+ \frac{1}{3\times 2}(x^3(2) + y^3(0) + 3x^2y(2) + 3xy^2(0))$

$x^2y + 3y - 2 = -10 + 2x^2y + x^2y + 3y + x^2y + 2yx^2 + \frac{x^3}{3} + x^2y + 0)$

$x^2y + 3y - 2 = -10 + 2x^2y + 2x^2y + 3y + 2x^2y + \frac{x^3}{3} + x^2y$

$\boxed{x^2y + 3y - 2 = -10 + 7x^2y + 3y + x^3/3}$

Not need to do

* Taylor's expansion in powers of h & k
Expand $\frac{(x+h)(y+k)}{x+h+y+k}$ up to second degree.

Sol:

$f(x+h,y+k) = f(x,y) + (hf_x + kf_y) + \frac{1}{2!}(h^2 f_{xx} + k^2 f_{yy} + 2hk f_{xy}) + \frac{1}{3!}(h^3 f_{xxx} + k^3 f_{yyy} + 3h^2k f_{xxy} + 3hk^2 f_{xyy}) + \dots$ —(1)

Put $h=k=0$

$$f(x+h,y+k) = \frac{(x+0)(y+0)}{x+y}$$

$$f(x+0,y+0) = \frac{xy}{x+y} \qquad \boxed{f(x,y) = \frac{xy}{x+y}}$$

$$f_x = \frac{(x+y)y - (xy)}{(x+y)^2} = \frac{\cancel{xy}+y^2-\cancel{xy}}{(x+y)^2} \qquad \boxed{f_x = \frac{y^2}{(x+y)^2}}$$

$$\boxed{f_y = \frac{x^2}{(x+y)^2}}, \quad \boxed{f_{xx} = \frac{(x+y)^2 0 - y^2 \cdot 2(x+y)}{(x+y)^4}}$$

$$f_{xx} = \frac{0 - y^2(2x+2y)}{(x+y)^4} \Rightarrow \boxed{f_{xx} = \frac{-2y^2\cancel{(x+y)}}{(x+y)^4}}$$

$$\boxed{f_{xx} = \frac{-2y^2}{(x+y)^3}} \qquad \boxed{f_{yy} = \frac{-2x^2}{(x+y)^3}}$$

$$\boxed{f_{xy} = \frac{2xy}{(x+y)^3}}$$

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$$\frac{(x+h)(y+k)}{x+h+y+k} = \frac{xy}{x+y} + h\left(\frac{y^2}{(x+y)^2}\right) + k\left(\frac{x^2}{(x+y)^2}\right) + \frac{1}{2!}\left[h^2\left(\frac{-2y^2}{(x+y)^3}\right) + 2hk\left(\frac{2xy}{(x+y)^3}\right) + k^2\left(\frac{-2x^2}{(x+y)^3}\right)\right] + \cdots$$

$$\frac{(x+h)(y+k)}{x+h+y+k} = \frac{xy}{x+y} + \frac{y^2}{(x+y)^2}h + \frac{x^2}{(x+y)^2}k - \frac{y^2}{(x+y)^3}h^2 + \frac{2xy}{(x+y)^3}hk - \frac{x^2}{(x+y)^3}k^2 + \cdots$$

Maxima & Minima

of function of two variable

steps :-

(i) $\boxed{\frac{\partial f}{\partial x} = 0}$ & , $\boxed{\frac{\partial f}{\partial y} = 0}$ or $f_x = 0$, $f_y = 0$

(ii) $\boxed{r = \frac{\partial^2 f}{\partial x^2}}$, $\boxed{s = \frac{\partial^2 f}{\partial x \partial y}}$, $\boxed{t = \frac{\partial^2 f}{\partial y^2}}$ or $r = f_{xx}$, $s = f_{xy}$, $t = f_{yy}$

(iii)
(i) if $rt - s^2 > 0$, $r > 0$; $f(x,y)$ is Minima
(ii) if $rt - s^2 > 0$, $r < 0$, $f(x,y)$ is Maxima
(iii) if $rt - s^2 < 0$, Neither Maxima nor minima
(iv) if $rt - s^2 = 0$, the case is doubtful, further investigation

* Discuss the maxima & minima of
$f(x,y) = x^2 + y^2 + 6x + 12$

$f_x = 0 \Rightarrow 2x + 6 = 0$ | $f_y = 0 \Rightarrow 2y = 0$

$\boxed{x = -3}$ | $\boxed{y = 0}$

$(-3, 0)$

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$s = f_{xy}$

$r = f_{xx}$ $\boxed{r = 2}$ $\boxed{s = 0}$ $\boxed{t = 2}$ $t = f_{yy}$

$rt - s^2 = 4 - 0 = 4$

$rt - s^2 > 0$, and $r > 0$ ($r = 2$)

There is minima at point $(-3, 0)$

Minimum value $f(x, y) = x^2 + y^2 + 6x + 12$

$f(-3, 0) = 9 + 6(-3) + 12$
$f(-3, 0) = 9 - 18 + 12$
$\boxed{f(-3, 0) = 3}$ minimum value is $\boxed{3}$

VI

* Examine the fun^n^ $f(x, y) = x^3 + y^3 - 3axy$ for maxima & minima.

Sol:

$f_x \Rightarrow 3x^2 - 3ay = 0$
$3x^2 = 3ay$
$3x^2 = a3y$
$\cancel{3}x^2 = \cancel{3}ay$
$\boxed{y = \frac{x^2}{a}}$ — (i)

$f_y \Rightarrow 3y^2 - 3ax = 0$
$y^2 = ax$ — (ii)
put (i) in eq^n^ (ii)

$\left(\frac{x^2}{a}\right)^2 = ax$

$\frac{x^4}{a^2} = ax \Rightarrow x^4 = a^3 x$

$= x \cdot x^3 = a \cdot x^3$
$x \cdot x^3 - a^3 x = 0$
$x(x^3 - a^3) = 0$
$\boxed{x = 0, a}$ — (iii)

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put (iii) value in eq^n (i)

$$y = \frac{x^2}{a}, \quad y = \frac{0}{a} \quad \boxed{y = 0}$$

$$y = \frac{a^2}{a} \quad \boxed{y = a}$$

$$\boxed{y = 0, a}, \quad x = 0, a$$

$\therefore$ (0,0), (a,a) are satationary point

Now $\quad r = f_{xx} \quad r = 6x$
$\quad s = f_{xy} \quad s = -3a$
$\quad t = f_{yy} \quad t = 6y$

$rt - s^2 \Rightarrow (6x)(6y) - 9a^2$
$= 36xy - 9a^2$
= At point (0,0)
$rt - s^2 = 0 - 9a^2$
$rt - s^2 = -9a^2$
$rt - s^2 < 0$ neither maxima and nor minima

again point At (a,a)

$rt - s^2 \Rightarrow 36xy - 9a^2$
$\Rightarrow 36a^2 - 9a^2$
$\Rightarrow 27a^2 \quad rt - s^2 = 27a^2$
$rt - s^2 > 0$ &
$r = 6x$
$r = 6a$ if a is positive
$r > 0$

There is minima at point (a,a)

if a is (-ve) then There $r < 0$ There is maxima at point (a,a)

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$\therefore$ minimum value :-
at point (a,a) $\Rightarrow x^3+y^3-3axy$

$\Rightarrow a^3+y^3-3a.a.a$
$\Rightarrow a^3+a^3-3a^3$
$= 2a^3-3a^3$
$= -a^3$

VVI

* Discuss maxima and minima of $x^3y^2(1-x-y)$.

ans $\Rightarrow$
$f(x,y) = x^3y^2(1-x-y)$
$f(x,y) = x^3y^2 - x^4y^2 - x^3y^3$

$f_x=0 \Rightarrow 3x^2y^2 - 4x^3y^2 - 3x^2y^3 = 0$ —— (i)

$f_y=0 \Rightarrow 2yx^3 - 2yx^4 - 3y^2x^3 = 0$ —— (ii)

from eqn (1) $3x^2y^2 - 4x^3y^2 - 3x^2y^3 = 0$
$x^2y^2(3-4x-3y) = 0$ — (2)

from eqn (ii) $2yx^3 - 2yx^4 - 3y^2x^3 = 0$
$yx^3(2-2x-3y) = 0$ —— (3)

again from eqn (2) $x^2y^2 = 0$ | $3-4x-3y=0$ —— (4)
$x=0$
$y=0$

again from eqn (3) $yx^3(2-2x-3y)=0$

$yx^3=0$ | $2-2x-3y=0$ —— (5)
$x=0$
$y=0$

Solving eqⁿ (4) & (5)

$3-4x-3y = 2-2x-3y = 0$

$3-2-4x+2x-\cancel{3y}+\cancel{3y} = 0$

$1-2x = 0$

$1 = 2x$

$\boxed{x = \frac{1}{2}}$ again putting x in eqⁿ (4)

$3 - \frac{4 \times 1}{2} - 3y = 0$

$1-3y = 0 \Rightarrow \boxed{y = \frac{1}{3}}$

∴ The satationary points are

$x=0$	$y=0$	$x = \frac{1}{2}$
$x=0$	$y=0$	$y = \frac{1}{3}$

∴ $r = f_{xx}$, $S = f_{xy}$, $t = f_{yy}$

$3x^2y^2 - 4x^3y^2 - 3x^2y^3$

$r = 6xy^2 - 12x^2y^2 - 6xy^3$

$s = 6x^2y - 8x^3y - 9x^2y^2$

$t = 2x^3 - 2x^4 - 6yx^3$

$\because rt - s^2 \Rightarrow$

$(6xy^2 - 12x^2y^2 - 6xy^3)(2x^3 - 2x^4 - 6yx^3) - (6x^2y - 8x^3y - 9x^2y^2)^2$

putting satationary points

at point $(0,0)$ $x=0$, $y=0$

$$\boxed{rt - s^2 = 0}$$

$\therefore$ further investication is required.

$\Delta f = f(a+h, b+k) - f(a,b)$

$\Rightarrow f(0+h, 0+k) - f(0,0) \Rightarrow f(h,k) - f(0,0)$

$\Rightarrow h^3k^2(1-h-k) - 0$

$\Rightarrow \underset{③}{h^3k^2} - \underset{①}{h^4k^2} - \underset{②}{h^3k^3}$

Note ① ② का power ज्यादा है $\therefore$ इसका value very less होगा। तो हमलोग ③ को लेंगे क्योंकी इसका power कम है।

h^3k^2 (neglecting $h^4k^2 - h^3k^3$)

$\Delta f = h^3k^2$ $\quad$ Δf is +ve if h is +ve

Δf is -ve if k is -ve

$\therefore$ The sign is changed with h.

$\therefore$ neither maxima nor minima

at point $\left(\frac{1}{2}, \frac{1}{3}\right)$ $\quad rt - s^2$

$\Rightarrow (6xy^2 - 12x^2y^2 - 6xy^3)(6x^3 - 2x^4 - 6yx^3) - [(6x^2y - 8x^3y - 9x^2y^2)]^2$

$\Rightarrow 6xy^2(1-2x-y)\,2x^3(1-x-3y) - [x^2y(6x - 8x^2 - 9xy^2)]^2$

$\Rightarrow 12x^4y^2(1-2x-y)(1-x-3y) - x^4y^2(6x-8x-9y)^2$

Put $x = \frac{1}{2}, y = \frac{1}{3}$

$\Rightarrow 12 \times \frac{1}{16} \times \frac{1}{9}\left(1 - \frac{1}{2}\times 2 - \frac{1}{3}\right)\left(1 - \frac{1}{2} - 3\times\frac{1}{3}\right) - \frac{1}{16}\times\frac{1}{9}\left(6 - 8\times\frac{1}{2} - 9\times\frac{1}{3}\right)^2$

$\Rightarrow \frac{1}{12}\left(-\frac{1}{3}\right)\left(-\frac{1}{2}\right) - \frac{1}{16\times 9}(-1)^2 \Rightarrow \frac{1}{36\times 2} - \frac{1}{16\times 9} = \frac{1}{72} - \frac{1}{144}$

$$\frac{2-1}{144} = \boxed{\frac{1}{144} > 0}$$

$$r = 6 \times \frac{1}{2} \times \frac{1}{3} - 12 \times \frac{1}{4} \times \frac{1}{9} - 6 \times \frac{1}{2} \times \frac{1}{27}$$

$$r = \cancel{\frac{1}{3}} - \cancel{\frac{1}{3}} - \frac{1}{9} \qquad \boxed{r = -\frac{1}{9}}$$

$\boxed{r < 0}$ There is maxima at point $x, y\left(\frac{1}{2}, \frac{1}{3}\right)$

Maximum value is

$$f(x,y) = x^3 y^2 (1 - x - y)$$

$$f\left(\tfrac{1}{2}, \tfrac{1}{3}\right) = \frac{1}{8} \times \frac{1}{9}\left(1 - \frac{1}{2} - \frac{1}{3}\right)$$

$$= \frac{1}{72}\left(\frac{1}{2} - \frac{1}{3}\right) \rightarrow \frac{1}{72}\left(\frac{3-2}{6}\right) \Rightarrow \frac{1}{72} \times \frac{1}{6}$$

$$= \frac{1}{432}$$

Ex = 3 Locate the stationary points of $x^4 + y^4 + 2x^2 + 4xy - 2y^2$ and determine their nature.

ans ⇒ $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$

$f_x = 0 \Rightarrow 4x^3 - 4x + 4y = 0$ — (i)

$f_y = 0 \Rightarrow 4y^3 + 4x - 4y = 0$ — (ii)

$x^3 - x + y = 0$ — (i)

$y^3 + x - y = 0$ — (ii)

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$x^3 - x + y = 0$ — (3)
$y^3 + x - y = 0$ — (4)

adding (3) + (4)

$x^3 - \cancel{x} + \cancel{y} = 0$
$y^3 + \cancel{x} - \cancel{y} = 0$
$+ \quad + \quad +$

$x^3 + y^3 = 0$

$\boxed{\because a^3 + b^3 = (a+b)(a^2+b^2-ab)}$

$(x+y)(x^2+y^2-xy) = 0$
$x+y=0$ या $x^2+y^2-xy=0$

$\boxed{x = -y}$ या $x^2+y^2-xy=0$

puting x in eq (1)

$(-y)^3 - (-y) + y = 0$

$-y^3 + y + y = 0$

$-y^3 + 2y = 0 \qquad 2y - y^3 = 0$

$y(2-y^2) = 0$

$\boxed{y=0}$ या $y^2 = 2$

$\boxed{y = \pm\sqrt{2}}$

$x = -y$

$\boxed{x=0} \Rightarrow x = -(+\sqrt{2}) \Rightarrow \boxed{x = -\sqrt{2}}$

$x = -(-\sqrt{2}) \Rightarrow \boxed{x = \sqrt{2}}$

$\boxed{x = 0, -\sqrt{2}, \sqrt{2}}$

$\boxed{y = 0, \sqrt{2}, -\sqrt{2}}$

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$r = 12x^2 - 4 \quad s = 4 \quad t = 12y^2 - 4$

$rt - s^2 = (12x^2 - 4)(12y^2 - 4) - 16$

at point $(\sqrt{2}, -\sqrt{2})$

$rt - s^2 = (12 \times 2 - 4)(12 \times 2 - 4) - 16$

$\Rightarrow (24 - 4)(24 - 4) - 16$

$\Rightarrow (20)(20) - 16$

$= \cancel{880 - 16}\ 400 - 16$

$= 384$

$rt - s^2 > 0$

$r = 12x^2 - 4 \quad | \quad r = 24 - 4$

$r = 12(\sqrt{2})^2 - 4 \quad | \quad r = 20$

$r > 0$

There is minima at point $(\sqrt{2}, -\sqrt{2})$

and minimum value is

$= (\sqrt{2})^4 + (-\sqrt{2})^4 - 2(\sqrt{2})^2 + 4\sqrt{2} \cdot \sqrt{2} - 2(\sqrt{2})^2$

$\Rightarrow \cancel{4 + 4 - 4} - 8 - \cancel{4}$

The minimum value is -8

at point $(0,0)$ $rt - s^2 = (12(0) - 4)(12(0) - 4) - 16$

$rt - s^2 = (-4)(-4) - 16$

$= 16 - 16$

$\boxed{rt - s^2 = 0}$

further investigation is requir

Ex = 4 Examine for minima and maxima value $\sin x + \sin y + \sin(x+y)$

$f(x,y) = \sin x + \sin y + \sin(x+y)$

$f_x = 0 \Rightarrow \cos x + \cos(x+y) = 0$ — (1)

$f_y = 0 \Rightarrow \cos y + \cos(x+y) = 0$ — (ii)

$r = -\sin x - \sin(x+y)$ — (3)

$s = -\sin(x+y)$ — (4)

$t = -\sin y - \sin(x+y)$.

subtracting eq (1) − (ii)

$$\cos x + \cancel{\cos(x+y)} = 0$$
$$\cos y + \cancel{\cos(x+y)} = 0$$

$\cos x - \cos y = 0$

$\cancel{\cos} x = \cancel{\cos} y$

$\boxed{x = y}$ → put in eq (1)

$\cos x + \cos(x+y) = 0$

$\cos x + \cos(2x) = 0$

$\cos 2x = -\cos x$

$\cancel{\cos} 2x = \cancel{\cos}(\pi - x)$

$2x = \pi - x$

$3x = \pi$

$\boxed{x = \frac{\pi}{3}}$ $\because$ $x = y$ $\therefore \left(x, y = \frac{\pi}{3}, \frac{\pi}{3}\right)$

The stationary point is $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$

$rt - s^2 \Rightarrow$

$\left(-\sin x - \sin(x+y)\right)\left(-\sin y - \sin(x+y)\right) - \sin^2(x+y)$

at point $\frac{\pi}{3}$

$\left(-\sin\frac{\pi}{3} - \sin\frac{2\pi}{3}\right)\left(-\sin\frac{\pi}{3} - \sin\left(\frac{2\pi}{3}\right)\right) - \sin^2\left(\frac{2\pi}{3}\right)$

$\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)^2$

$(-\sqrt{3})(-\sqrt{3}) - \frac{3}{4} \Rightarrow 3 - \frac{3}{4} = \frac{9}{4}$

$r < 0 \mid rt - s^2 > 0$

~~There is maxima~~

$r = -\sin x - \sin(x+y)$

$r = -\sin\frac{\pi}{3} - \sin\frac{2\pi}{3}$

$r = -\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \qquad r = -\frac{2\sqrt{3}}{2}$

$r = -\sqrt{3}$

$r < 0$

There is maxima and maximum value is

$\sin\frac{\pi}{3} + \sin\frac{\pi}{3} + \sin\left(\frac{2\pi}{3}\right)$

$\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

Ex=6 A rectangular box, open at the top, is to have a volume of 32 cubic feet. find the dimensions of the box requiring least material for its construction.

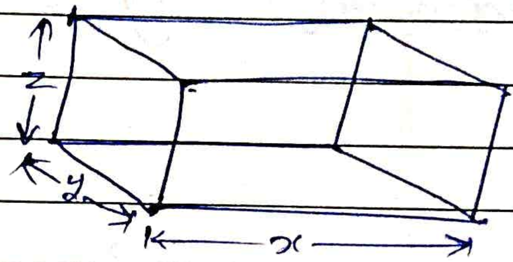
$V = 32$ cf

ans⇒ given V = volume

let x = length

y = breadth

z = height



$V = xyz$ | (∵ आयताकार = घनाभ का आयतन (xyz))

$xyz = V$

$\boxed{xyz = 32}$ $\boxed{z = \frac{32}{xy}}$ — (i)

$S = xy + 2yz + 2zx$ — (ii)

Put (i) in (ii)

$S = xy + 2y \times \frac{32}{xy} + 2x \times \frac{32}{xy}$

$\boxed{S = xy + \frac{64}{x} + \frac{64}{y}}$ — (iii)

$f_x = 0 \Rightarrow f_x = y + 64\left(\frac{-1}{x^2}\right)$ | $f_y = 0 \Rightarrow x - \frac{64}{y^2}$

$f_x \Rightarrow y - \frac{64}{x^2} = 0$ — (1) | $f_{xx} = \frac{-64(-2)}{x^3}$ $\boxed{f_{xx} = \frac{128}{x^3}}$

$f_y \Rightarrow x - \frac{64}{y^2} = 0$ — (ii) | $\boxed{f_{yy} = \frac{128}{y^3}}$ $f_{xy} = 1$

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$$y - \frac{64}{x^2} = 0 \quad \text{--- (1)}$$

$$x - \frac{64}{y^2} = 0 \quad \text{--- (2)}$$

from eqⁿ (1) $y = \frac{64}{x^2}$ $\quad yx^2 = 64$ --- (3)

$$\boxed{y = \frac{64}{x^2}} \text{ --- (5)}$$

$$xy^2 = 64 \quad \text{--- (4)}$$

Put eqⁿ (5) in (4)

$$x\left(\frac{64}{x^2}\right)^2 = 64$$

$$x \cdot \frac{64 \times 64}{x^4} = 64 \Rightarrow \frac{x \times 64}{x^4} = \cancel{\frac{64}{64}}$$

$$\cancel{x} \cdot 64 = x^4$$

$$x^3 = 64 \Rightarrow x = \sqrt[3]{64}$$

$\boxed{x = 4}$ put in (5)

$$y = \frac{64}{16} \quad \boxed{y = 4}$$

The stationary points are x, y $(4, 4)$

$$rt - s^2 = \left(\frac{128}{x^3}\right)\left(\frac{128}{y^2}\right) - 1$$

$$= \frac{\cancel{128}^2}{\cancel{64}} \times \frac{\cancel{128}^2}{\cancel{64}} - 1$$

$$= 4 - 1$$

$$= 3 \qquad rt - s^2 > 0$$

$$r = \frac{\cancel{128}^2}{\cancel{64}}$$

$$r = 2$$

$$r > 0$$

There is minima at (4, 4)

The minimum value is (8)

∴ Dimemsitions of the box is

$x = 4$

$y = 4$

$z = \frac{32}{xy} \Rightarrow z = \frac{32}{16}$

$\boxed{z = 2}$ $(x, y, z) = (4, 4, 2)$

E=7 :

* prove that if the perimeter of a triangle is constant, its area is maximum when the triangle is equilateral.

ans ⇒ $s = \frac{a+b+c}{2}$ There 2s is constant

$2s = a+b+c$

$\boxed{c = 2s-a-b}$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{s(s-a)(s-b)(s-(2s-a-b))}$

$\Delta = \sqrt{s(s-a)(s-b)(s-2s+a+b)}$

$\Delta = \sqrt{s(s-a)(s-b)(-s+a+b)}$

$\Delta = \sqrt{s(s-a)(s-b)(a+b-s)}$

Square both sides

$f = \Delta^2 = s(s-a)(s-b)(a+b-s)$

$\boxed{f = s(s-a)(s-b)(a+b-s)}$

$f_a = s(s-b)[(s-a)(a+b-s)]$

$f_a = s(s-b)[(s-a)(1)+(a+b-s)\cdot(-1)]$

$f_a = s(s-b)[\cancel{s}-a-a-b+\cancel{s}]$

$f_a = s(s-b)(-2a-b+2s)$

$\boxed{f_a = s(s-b)(-2a-b+2s)}$

$\boxed{f_a = s(s-b)(2s-2a-b)}$

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$f = s(s-a)(s-b)(a+b-s)$

$f_b = s(s-a)[(s-b)(a+b-s)]$

$f_b = s(s-a)[(s-b)(1)+(a+b-s)(-1)]$

$f_b = s(s-a)[s-b-a-b+s]$

$\boxed{f_b = s(s-a)(2s-2b-a)}$

$r = f_{aa} = s(s-b)(-2)$

$\boxed{f_{aa} = -2s(s-b)}$

$f_{ab} = s(s-b)(2s-2a-b)$

$f_{ab} = s[(s-b)(-1)+(2s-2a-b)(-1)]$

$s = f_{ab} = s(s-b(-1)+(-2s+2a+b)]$

$s = s(-s+b-2s+2a+b)]$

$s = s(-2s+2b+2a)$

$\boxed{s = s(2a+2b-2s)}$

$\boxed{s = s(2a+2b-3s)}$

$t = f_{bb} = s(s-a)(-2)$

$\boxed{t = -2s(s-a)}$

$f_a = 0 \Rightarrow s(s-b)(2s-2a-b) = 0$

$s(s-b)(2s-2a-b) = 0$

$s-b=0$ या $2s-2a-b=0$

$\boxed{s=b}$

$2s-2a-b=0$

$2s-2b-b=0$

$2s = 3b$

$\boxed{b = \frac{2s}{3}}$

$\because a=b$

$f_b = 0 \Rightarrow s(s-a)(2s-2b-a)$

$\Rightarrow s-a=0$ या $2s-2b-a=0$

$\boxed{s=a}$

$\therefore s=a=b$ $\boxed{a=b}$

$s=a=b$

$2s-2b-b=0$

$2s = 3b$

$\boxed{b = \frac{2s}{3}}$

$\therefore a = \frac{2s}{3}, b = \frac{2s}{3}$

$c = 2s - a - b$

$$\boxed{\begin{aligned} c &= 2s - a - b \\ c &= 2s - 2b - a \end{aligned}} \quad \Big\} \because s = a = b$$

$$\left.\begin{aligned} c &= 2s - a - b \\ c &= 2s - \frac{2s}{3} - \frac{2s}{3} \end{aligned}\right\} \text{puting } a = \frac{2s}{3}$$

$c = 2s - \frac{4s}{3}$ या or $c = \frac{6s - 4s}{3}$

$$\boxed{c = \frac{2s}{3}}$$

This implies $a = b = c = \frac{2s}{3}$

Hence prove it is equilateral triangle

$r = s(s-b)(-2)$ $\boxed{r = -2s(s-b)}$

$r = -2s\left(s - \frac{2s}{3}\right)$ or $r = -2s\left(\frac{3s - 2s}{3}\right)$

$r = -2s \times \frac{s}{3}$ $\boxed{r = \frac{-2s^2}{3}}$

$s = s\left(\frac{2 \times 2s}{3} + \frac{2 \times 2s}{3} - 3s\right)$

$s = s\left(\frac{4s}{3} + \frac{4s}{3} - 3s\right) \Rightarrow s = s\left(\frac{8s}{3} - \frac{3s}{1}\right)$

$s = s\left(\frac{8s - 9s}{3}\right) \Rightarrow s = \frac{s(-s)}{3}$ $\boxed{s = \frac{-s^2}{3}}$

$t = -2s\left(s - \frac{2s}{3}\right)$ $t = -2s\left(\frac{3s - 2s}{3}\right)$

$\boxed{t = \frac{-2s^2}{3}}$ $rt - s^2 =$

$\Rightarrow \left(\frac{-2s^2}{3}\right)\left(\frac{-2s^2}{3}\right) - \frac{s^4}{9} \Rightarrow \frac{4s^4}{9} - \frac{s^4}{9} = \frac{3s^4}{9} = \frac{s^4}{3}$

$\boxed{\frac{s^4}{3} > 0}$

cheak:-

$$r = -2s(s-b)$$

$$\boxed{r = \frac{-2s^2}{3}} \qquad r < 0$$

There is maxima

* Tangent & Normal

$f(x, y, z) = 0$ is eqn of surface

then find $\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$ at point (x_1, y_1, z_1)

The eqn of normal line is given by

$$\boxed{\frac{x - x_1}{\frac{\partial f}{\partial x}} = \frac{y - y_1}{\frac{\partial f}{\partial y}} = \frac{z - z_1}{\frac{\partial f}{\partial z}}}$$

$$\boxed{\frac{\partial f}{\partial x}(x - x_1) + \frac{\partial f}{\partial y}(y - y_1) + \frac{\partial f}{\partial z}(z - z_1) = 0}$$

$$\boxed{f_x(x - x_1) + f_y(y - y_1) + f_z(z - z_1) = 0}$$

* Find the equation of the tangent plane for the surface $x^3 + y^3 + 3xyz = 3$ at $(1, 2, -1)$

ans ⇒ $f(x, y, z) = x^3 + y^3 + 3xyz = 3$

$\Rightarrow x^3 + y^3 + 3xyz - 3 = 0$

$f_x = 3x^2 + 3yz$
$f_y = 3y^2 + 3xz$
$f_z = 3xy$

at point $(1, 2, -1)$

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$\cancel{f(x)}$ $fx = 3x^2 + 3yz$

$fx = 3(1)^2 + 3(2)(-1)$

$fx = 3 - 6$

$\boxed{fx = -3}$

$fz = 3(1)(2)$

$\boxed{fz = 6}$

$fy = 3(2)^2 + 3(1)(-1)$

$fy = 12 - 3$

$\boxed{fy = 9}$

$\therefore$ The equation of tangent plane is

$fx(x - x1) + fy(y - y1) + fz(z - z1) = 0$

$-3(x-1) + 9(y-2) + (6)(z - (-1)) = 0$

$-3x + 3 + 9y - 18 + 6z + 6 = 0$

$-3x + 9y + 6z - \cancel{18} = 0$

$-3x + 9y + 6z - 9 = 0$

$3x - 9y - 6z + 9 = 0$

$\boxed{x - 3y - 2z + 3 = 0}$

* Find the equation of tangent plane to the surface $\frac{x^2}{2} + \frac{y^2}{2} = z$ at point $(2, 3, -1)$

ans ⇒ Given : $\frac{x^2}{2} + \frac{y^2}{2} = z$

$\frac{x^2 + y^2}{2} = z \Rightarrow x^2 + y^2 = 2z$

$\boxed{x^2 + y^2 - 2z = 0}$

$f_x = 2x$ at $(2,3,-1)$
$f_y = 2y$
$f_z = -2$

$f_x = 4$
$f_y = 6$
$f_z = -2$

$\therefore$ The eqⁿ of tangent plane is

$$f_x(x-x_1)+f_y(y-y_1)+f_z(z-z_1)=0$$

$$4(x-2)+6(y-3)+(-2)(z+1)=0$$

$$4x-8+6y-18-2z-2=0$$
$$4x+6y-2z-28=0$$
$$2(2x+3y-z-14)=0$$

$$\boxed{2x+3y-z-14=0}$$

* $\frac{x^2}{2}-\frac{y^2}{3}=z$ at $(2,3,-1)$

$\frac{3x^2-2y^2}{6}=z$ or $3x^2-2y^2=6z$

$$3x^2-2y^2-6z=0$$

$f_x = 6x$
$f_y = -4y$
$f_z = -6$

at point $(2,3,-1)$

$f_x = 12$
$f_y = -12$
$f_z = -6$

The eqⁿ of tangent plane

$$f_x(x-x_1)+f_y(y-y_1)+f_z(z-z_1)=0$$

$$12(x-2)+(-12)(y-3)+(-6)(z+1)=0$$

NOTES

$12x-24-12y+36-6z-6=0$

$12x-12y-6z+6=0$

$6(2x-2y-z+1)=0$

$\boxed{2x-2y-z+1=0}$

* $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{25}=1$ at $(2,3,5)$

ans ⇒ $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{25}-1=0$ at point $(2,3,5)$

$f_x=\frac{1}{4}\times 2x=\frac{x}{2} \Rightarrow \frac{2}{2}=1$

$f_y=\frac{1}{9}\times 2y=\frac{2y}{9} \Rightarrow \frac{2\times 3}{9}=\frac{2}{3}$

$f_z=-\frac{1}{25}\times 2z=-\frac{2}{25}z \Rightarrow -\frac{2}{25}\times 5=-\frac{2}{5}$

The eqn of tangent line

$(x-2)+\frac{2}{3}(y-3)-\frac{2}{5}(z-5)=0$

$x-2+\frac{2y}{3}-2-\frac{2z}{5}+2=0$

$\boxed{x-\frac{2y}{3}-\frac{2z}{5}-2=0}$

NOTES

* $2x^2+y^2+2z=3$ at $(2,1,-3)$

$f_x = 4x$ | $f_x = 8$
$f_y = 2y$ | $f_y = 2$
$f_z = 2$ | $f_z = 2$

The eq^n of tangent plane

$8(x-2)+2(y-1)+2(z+3)$

$8x-16+2y-2+2z+6$

$8x+2y+2z-12=0$

$2(4x+y+z-6=0)$

$$\boxed{4x+y+z-6=0}$$

* what is the condition for (x, y) to be maxima or minima.

Sol^n: minima ~~maxima~~ :

If $rt-s^2>0$, $r>0$ then $f(x, y)$ is minima.

maxima :

If $rt-s^2>0$, $r<0$ then $f(x, y)$ is maxima.

where $\boxed{r=f_{xx}}$, $\boxed{t=f_{yy}}$, $\boxed{s=f_{xy}}$